MATH 360 HOMEWORK 2 SOLUTIONS

Chapter 0

1) **#49** $S = \mathbb{Z}$, aRb iff $ab \geq 0$.

Is R an equivalence relation?

No. Because it is not transitive

$3R0$ b/c $3 \cdot 0 = 0 \geq 0$

$0R{-2}$ b/c $0 \cdot -2 = 0 \geq 0$

but $3 \not R {-2}$ b/c $3 \cdot -2 = -6 \not\geq 0$

#50. $S = \mathbb{Z}$, aRb iff $a+b$ is even

i) $a+a = 2a$ is even so aRa (reflexive)

ii) $aRb \Rightarrow a+b = 2k$ for some $k \in \mathbb{Z}$

Then $b+a = 2k$ as well $\Rightarrow bRa$ (symmetric)

iii) aRb and $bRc \Rightarrow a+b = 2k_1$ for some $k_1 \in \mathbb{Z}$

and $b+c = 2k_2$ for some $k_2 \in \mathbb{Z}$

Then $a+c = 2k_1 - b + 2k_2 - b = 2(k_1 - b + k_2)$

$\Rightarrow aRc$ (transitive)

So, R is an equivalence relation

Equivalence classes: $\bar{0} = \{0, \mp 2, \mp 4, \mp 6, \dots\}$

$\bar{1} = \{\mp 1, \mp 3, \mp 5, \dots\}$

#52. Prove that none of $11, 111, 1111, \dots$ is a square.

Note that $\underbrace{11\dots11}_{n} = \underbrace{10^{n-1} + 10^{n-2} + \dots + 100}_{\text{divisible by } 4} + \underbrace{10+1}_{\text{remainder } 3 \text{ when divided by } 4}$

Let $n \in \mathbb{Z}$. There are two cases:

n is even $\Rightarrow n = 2k \Rightarrow n^2 \equiv 0 \mod 4$

n is odd $\Rightarrow n = 2k+1 \Rightarrow n^2 = 4k^2 + 4k + 1 \equiv 1 \mod 4$

So square of no integer gives a remainder of 3 when divided by 4.

2) a) $f: A \to B$, $g: B \to C$

$g \circ f$ 1-1 $\Rightarrow f$ is 1-1

Proof: Suppose $f(x) = f(y)$ for some $x, y \in A$

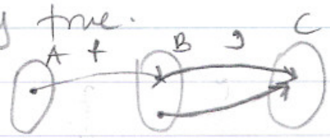
Then $(g \circ f)(x) = (g \circ f)(y)$

Since $g \circ f$ is 1-1, $x = y$.

b) $g \circ f$ 1-1 $\Rightarrow g$ is 1-1

not necessarily true.

example:



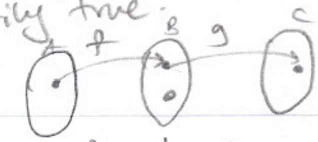
g not 1-1

but $g \circ f$ is

c) $g \circ f$ onto $\Rightarrow f$ onto.

not necessarily true.

example:



f not onto

$g \circ f$ is onto.

d) $g \circ f$ onto $\Rightarrow g$ onto.

Proof: Let $z \in C$.

Since $g \circ f$ is onto, $\exists x \in A$ s.t. $(g \circ f)(x) = z$.

Then $g(y) = z$ where $y = f(x)$.

So, $\exists y \in B$ s.t. $g(y) = z$.

Hence g is onto.

3) **Chapter 2 #1**

Two reasons why $\{\mp 1, \mp 3, \dots\}$ is not a group under $+$?

1) not closed: $1+3 = 4$ not odd

2) no identity.

4) $\mathbb{Z}_5 = \{\bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}\}$

Define $*$ on $\mathbb{Z}_5$ as

$\bar{a} * \bar{b} = \overline{ab}$

Show that this operation is well defined

Proof

let $x \in \bar{a} \Rightarrow 5 \mid x-a$

$y \in \bar{b} \Rightarrow 5 \mid y-b$

Goal Show that

$xy \in \overline{ab}$

$x = 5k_1 + a$

$y = 5k_2 + b$

$\Rightarrow xy = (5k_1 + a)(5k_2 + b)$

$= 25k_1k_2 + 5k_1b + 5k_2a + ab$

$xy = 5(5k_1k_2 + k_1b + k_2a) + ab$

$5 \mid xy - ab \Rightarrow xy \in \overline{ab}$ ∎