MATH 149 CALCULUS I

SOLUTIONS TO HOMEWORK 11

2.6.18. The radius r increasing at a rate of 2 in/min
Find the rate of change of volume when r = 6 in
r = 24 in

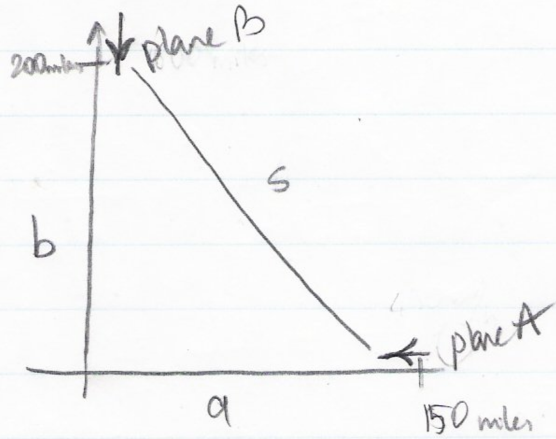
$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3} r^2 \frac{dr}{dt}$$

given $\frac{dr}{dt} = 2$ in/min so $\frac{dV}{dt} = 8\pi r^2$ in^3/min

at $r = 6$, $\frac{dV}{dt} = 8 \cdot \pi \cdot 6^2 = 8\pi \, 36 = 288\pi$ in^3/min

at $r = 24$, $\frac{dV}{dt} = 8\pi \cdot 24^2 = 8 \cdot \pi \cdot 576$ in^3/min
$= 4608\pi$ in^3/min

2.6.31.



s = distance between two planes at time t

a = distance of plane A from the intersection

b = distance of plane B "

Given:

$\frac{da}{dt} = -450$ miles/hour , $\frac{db}{dt} = -600$ miles/hour

a) To be determined: $\frac{ds}{dt}$

Equation: $s^2 = a^2 + b^2$

take derivative with respect to time $2s\frac{ds}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt}$

$$\frac{ds}{dt} = \frac{2a.(-450) + 2b(-600)}{2s}$$

So, $\frac{ds}{dt} = \frac{a}{s}(-450) + \frac{b}{s}(-600)$

at the point $a = 150$, $b = 200$,
we have $s = \sqrt{150^2+200^2} = 250$

So, $\frac{ds}{dt} = \frac{150}{250}(-450) + \frac{200}{250}(600)$

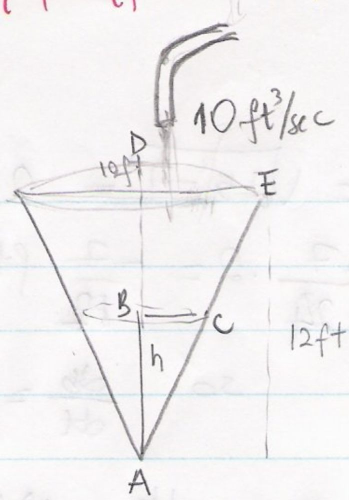
$= -270 + (-480)$

$= -750$ miles/hour.

b) 250 miles and decreasing by -750 miles/hour

$\Rightarrow$ he has 20 minutes!

2.6.24.



$$\frac{dV}{dt} = 10\ ft^3/sec$$

$$\frac{dh}{dt} = ? \text{ when } h = 8\ ft.$$

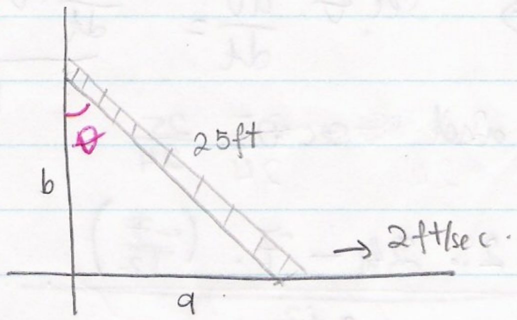
$$V = \frac{\pi r^2 h}{3}$$

$$\triangle ABC \sim \triangle ADE \Rightarrow \frac{h}{12} = \frac{r}{5} \Rightarrow r = \frac{5h}{12}$$

So,
$$V = \frac{\pi \cdot \left(\frac{5h}{12}\right)^2 \cdot h}{3} \Rightarrow V = \frac{25\pi}{144 \cdot 3} h^3$$

$$\underbrace{\frac{dV}{dt}}_{10} = \frac{25\pi}{432} \underbrace{3h^2}_{8} \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{10 \cdot 432}{25\pi \cdot 3 \cdot 8^2} = \frac{9}{10\pi}\ ft/sec$$

2.6.27.



$$\frac{da}{dt} = 2\ ft/sec$$

$$\frac{db}{dt} = ?$$

Pythagorean theorem:
$$a^2 + b^2 = 25^2$$
$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$
$$\frac{db}{dt} = -\frac{a}{b}\frac{da}{dt}$$

when $a = 7\text{ ft}$, $b = \sqrt{25^2 - 7^2} = 24\text{ ft}$

so $\frac{db}{dt} = \frac{-7}{24} \cdot 2 = \frac{-7}{12}$ ft/sec

$a = 15\text{ ft} \Rightarrow b = 20\text{ ft}$. so $\frac{db}{dt} = \frac{-15}{20} \cdot 2 = \frac{-3}{2}$ ft/sec

$a = 24\text{ ft} \Rightarrow b = 7\text{ ft}$ so $\frac{db}{dt} = \frac{-24}{7} \cdot 2 = -\frac{48}{7}$ ft/sec

b) area of the triangle $= A = a \cdot b$

$$\Rightarrow \frac{dA}{dt} = \frac{da}{dt} \cdot b + a \cdot \frac{db}{dt}$$

$a = 7 \Rightarrow b = 24$ and $\frac{db}{dt} = \frac{-7}{12}$ ft/sec

So, $\frac{dA}{dt} = 2 \cdot 24 + 7 \cdot \frac{-7}{12} = \frac{527}{24}$ ft^2/sec.

c) $\tan\theta = \frac{a}{b} \Rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \dfrac{\frac{da}{dt} \cdot b - a \cdot \frac{db}{dt}}{b^2}$

when $a = 7$, $b = 24$ and $\sec\theta = \frac{25}{24}$

$$\text{so, } \left(\frac{25}{24}\right)^2 \frac{d\theta}{dt} = \frac{2 \cdot 24 - 7 \cdot \left(\frac{-7}{12}\right)}{24^2}$$

$$\frac{d\theta}{dt} = \frac{1}{12} \text{ radians/sec}$$

Section 3.1

#17. Find critical numbers of $h(x) = \sin^2 x + \cos x$ for $0 < x < 2\pi$

$h'(x) = 2\sin x \cdot \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0 \Rightarrow \sin x = 0 \qquad 2\cos x - 1 = 0$

$x = \pi, \qquad \cos x = \frac{1}{2}$

$h'(x)$ is defined for all $x \in (0, 2\pi)$ $\qquad x = \frac{\pi}{3}, \frac{5\pi}{3}$

so only critical points in $(0, 2\pi)$ are $x = \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

#28. Locate absolute extrema of $f(x) = \dfrac{2x}{x^2+1}$ on $[-2, 2]$

*) critical numbers.

$$f'(x) = \frac{2(x^2+1) - 2x(x^2+1)}{((x^2+1))^2} = \frac{-2(x^3 - x^2 + x - 1)}{(x^2+1)^2} = \frac{-2(x-1)^2(x+1)}{(x^2+1)^2}$$

$f'(x) = 0 \Rightarrow x = 1, \ x = -1 \to$ critical numbers.

$f'(x)$ is defined for all x in $[-2, 2]$ so no critical numbers from there.

**) Evaluate at critical numbers and end points.

critical numbers	$f(-1) = \frac{-2}{2} = -1 \to$ absolute minimum	
	$f(1) = \frac{2}{2} = 1 \to$ absolute maxim.	
end points	$f(-2) = \frac{-4}{5}$	
	$f(2) = \frac{4}{5}$	

#40. $f(x) = \sqrt{4 - x^2}$ $\qquad$ Critical numbers: $f'(x) = \dfrac{-2x}{2\sqrt{4-x^2}} = \dfrac{x}{\sqrt{4-x^2}}$

$f'(x)$ does not exist when $x = 2, -2$ (in the domain)

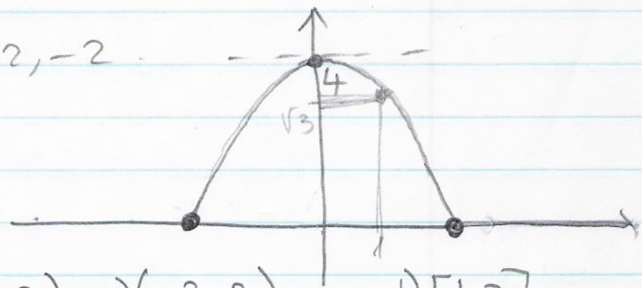
$f'(x) = 0$ when $x = 0$

Critical numbers: $x = 0, 2, -2$

$f(0) = \sqrt{4} = 2$

$f(2) = \sqrt{4-4} = 0$

$f(-2) = \sqrt{4-4} = 0$



	a) $[-2, 2]$	b) $[-2, 0)$	c) $(-2, 2)$	d) $[1, 2]$
absolute maximum	2 at $x=0$	does not exist	2 at $x=0$	$\sqrt{3}$ at $x=1$
absolute minimum	0 at $x=2$ and $x=-2$	0 at $x=-2$	does not exist.	0 at $x=2$.

#60. Cost $= C(x) = 2x + \dfrac{300{,}000}{x}$ $\qquad 1 \le x \le 300$.

critical numbers $\qquad C'(x) = 2 - \dfrac{300{,}000}{x^2} = 0 \Rightarrow x \cong 387$ not in $[1, 300]$

$C(1) = 300{,}002$

$\boxed{C(300) = 1600}$ minimum

minimum cost of \$1600 at $x = 300$

If $1 \le x \le 400$, then compare

$C(1) = 300{,}002$, $\boxed{C(387) \cong 1549.19}$ minimum $\quad C(400) = 1550$