MATH 16a Homework 12 Solutions

Sec 3.2 #22. $f(x) = \cos(2x)$ → continuous on $\left[-\frac{\pi}{12}, \frac{\pi}{6}\right]$, differentiable on $\left(-\frac{\pi}{12}, \frac{\pi}{6}\right)$

$f\left(-\frac{\pi}{12}\right) \neq f\left(\frac{\pi}{6}\right)$, with $f\left(-\frac{\pi}{12}\right) = \frac{\sqrt{3}}{2}$ and $f\left(\frac{\pi}{6}\right) = \frac{1}{2}$

so Rolle's Theorem does not apply.

#38. $f(x) = -x^2 - x + 6$

slope of secant line through $(-2,4)$ and $(2,0)$ $= \frac{0-4}{2-(-4)} = -1$

equation: $y = -1(x-2) \Rightarrow y = -x+2$

f is continuous on $[-2,2]$ and differentiable on $(-2,2)$ so by the M.V.T. there is a number c in $(-2,2)$ where

$f'(c) = \underbrace{-1}_{\text{slope of secant}} \Rightarrow -2x - 1 = -1 \Rightarrow x = 0$

$f(0) = 6 \Rightarrow$ equation of the tangent line: $y - 6 = -1(x-0)$ $\boxed{y = -x+6}$

#42. $f(x) = \frac{x+1}{x}$ on $\left[\frac{1}{2}, 2\right]$

M.V.T applies b/c f is continuous on $[1/2, 2]$ and differentiable on $(1/2, 2)$

slope of the secant line: $\frac{f(2) - f(1/2)}{2 - \frac{1}{2}} = -1$

$f'(x) = \frac{x-(x+1)}{x^2} = -\frac{1}{x^2} \longrightarrow \frac{-1}{x^2} = -1 \Rightarrow x = \pm 1$

$\boxed{x = 1}$ is in $(1/2, 1)$

#48. $f(x) = x - 2\sin x$ on $[-\pi, \pi]$

M.V.T. applies b/c f is continuous on $[-\pi, \pi]$ and differentiable on $(-\pi, \pi)$

slope of the secant line: $\frac{f(\pi) - f(-\pi)}{\pi - (-\pi)} = 1$

$f'(x) = 1 - 2\cos x$ $\qquad 1 - 2\cos x = 1$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}$, $x = -\frac{\pi}{2}$

There are two tangent lines parallel to the secant line:

1) at $x = \frac{\pi}{2}$, $y = \frac{\pi}{2} - 2 \Rightarrow y - \left(\frac{\pi}{2} - 2\right) = 1 \cdot \left(x - \frac{\pi}{2}\right) \Rightarrow \boxed{y = x - 2}$

at $x = -\frac{\pi}{2}$, $y = -\frac{\pi}{2} + 2 \Rightarrow y - \left(-\frac{\pi}{2} + 2\right) = 1 \cdot \left(x + \frac{\pi}{2}\right) \Rightarrow \boxed{y = x + 2}$

 # 52. $S(t) = 200\left(5 - \frac{9}{2+t}\right)$: t: months

a) average value during first year (12 months) $\frac{f(12) - f(0)}{12 - 0} = \frac{450}{7}$

b) $S'(t) = 200 \cdot \frac{9}{(2+t)^2} = \frac{450}{7} \Rightarrow \frac{1}{(2+t)^2} = \frac{1}{28} \Rightarrow 2 + t = \mp 2\sqrt{7}$

$\Rightarrow t = 2\sqrt{7} - 2$

or $t = -2\sqrt{7} - 2$ negative

$\boxed{\Rightarrow t = 2\sqrt{7} - 2 \cong 3.3 \text{ months} \to \text{in April}}$