MATH 150 HOMEWORK 6 SOLUTIONS

2.1 #14. $f(x) = 3x+2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)+2-(3x+2)}{h} = \lim_{h\to 0} \frac{3x+3h+2-3x-2}{h}$$

$$= \lim_{h\to 0} \frac{3h}{h} = 3.$$

#18. $f(x) = 1-x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{1-(x+h)^2-(1-x^2)}{h} = \lim_{h\to 0} \frac{1-(x^2+2xh+h^2)-1+x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{1}-\cancel{x^2}-2xh-h^2-\cancel{1}+\cancel{x^2}}{h} = \lim_{h\to 0} \frac{\cancel{h}(-2x-h)}{\cancel{h}} = \lim_{h\to 0} -2x-h = -2x.$$

#30. $f(x) = \sqrt{x-1}$, point of tangent $(5,2)$

$$\text{slope} = f'(5) = \lim_{h\to 0} \frac{f(5+h)-f(5)}{h} = \lim_{h\to 0} \frac{\sqrt{5+h-1}-2}{h} \cdot \frac{(\sqrt{h+4}+2)}{(\sqrt{h+4}+2)}$$ (rationalizing technique)

$$= \lim_{h\to 0} \frac{h+\cancel{4}-\cancel{4}}{h(\sqrt{h+4}+2)} = \lim_{h\to 0} \frac{1}{\sqrt{h+4}+2} = \frac{1}{4} = \text{slope}.$$ Tangent line: $y-2 = \frac{1}{4}(x-5)$

(sketch on DERIVE) $\boxed{y = \frac{1}{4}x + \frac{3}{4}}$

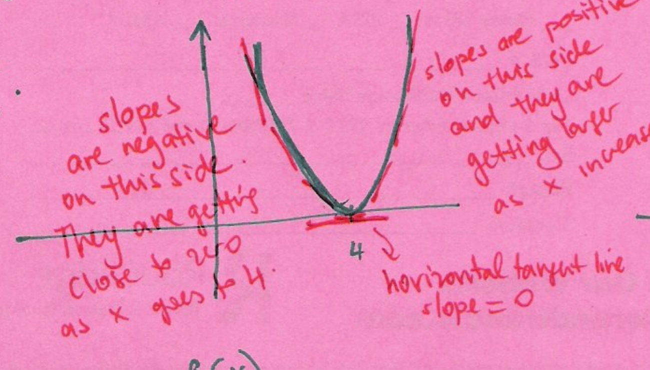
#32. $f(x) = \frac{1}{x+1}$, point of tangent $(0,1)$

$$\text{slope} = f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{\frac{1}{h+1}-1}{h} = \lim_{h\to 0} \frac{\frac{1-h-1}{h+1}}{h} =$$

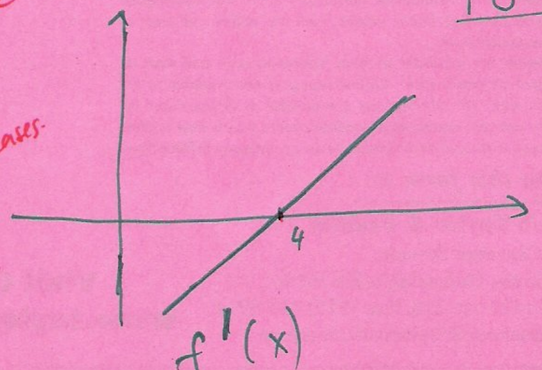
$$\lim_{h\to 0} \frac{-h}{h+1}\cdot\frac{1}{h} = \lim_{h\to 0} \frac{-1}{h+1} = -1 = \text{slope}.$$ Tangent line: $y-1 = -1(x-0)$

$\boxed{y = -x+1}$

#45.



f(x)



f'(x)