Section 2.2 #92. $f(x) = \sin x$ on $[0, \frac{\pi}{6}]$

average rate of change $= \dfrac{\sin\frac{\pi}{6} - \sin 0}{\frac{\pi}{6} - 0} = \dfrac{\frac{1}{2} - 0}{\frac{\pi}{6}} = \dfrac{3}{\pi} = \boxed{.95}$

instantaneous rate of change $= (\sin x)' = \cos x$

at $x = 0$, $\cos(0) = \boxed{1}$

at $x = \frac{\pi}{6}$, $\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} = \boxed{.866}$

close to the average of the two instanteous rates of change.

#94.

220 ft

initial velocity $= -22$ ft/sec

initial height $= 220$ ft

position $= s(t) = -16t^2 - 22t + 220$

velocity $= v(t) = s'(t) = -32t - 22$

after 3 seconds velocity $= v(3) = -32 \cdot 3 - 22 = -118$ ft/sec

When falls 108 ft, height from ground is $220 - 108 = 112$ ft

To find the time it takes to fall 108 ft solve

$108 = -16t^2 - 22t + 220$ for t

$\Rightarrow 16t^2 + 22t - 112 = 0 \Rightarrow \boxed{t = 2}$ or $t = \frac{-27}{8}$ (using quadratic formula)

velocity at 2 seconds $= v(2) = -32 \cdot (2) - 22$

$= -86$ ft/sec.

#96. drop the stone $\Rightarrow$ initial velocity $= 0$.

$s(t) = -4.9t^2 + h_0$ (h_0 initial height unknown)

It takes 6.8 seconds to hit the pool

$\Rightarrow s(6.8) = 0$ (height is zero when it hits the pool)

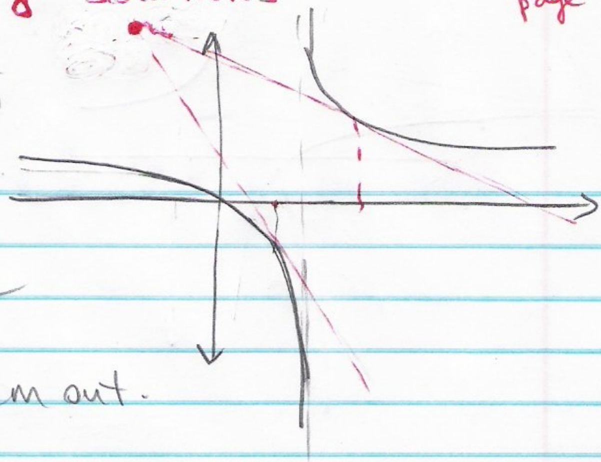
$-4.9(6.8)^2 + h_0 = 0 \Rightarrow \boxed{h_0 = 226.576 \text{ meters}}$

#106. Inventory cost $= C = \dfrac{1{,}008{,}000}{Q} + 6.3Q$ (Q: order size)

$C' = -\dfrac{1{,}008{,}000}{Q^2} + 6.3 \Rightarrow$ instantenous rate of change when $Q = 350$ $= \dfrac{-1{,}008{,}000}{350^2} + 6.3 \cong -1.93$

change in C: $C(351) - C(350) = -1.91$ — the two numbers are very close.

Section 2.3 #78. $f(x) = \frac{x}{x-1}$



According to the picture, there are two tangent lines.

We do NOT know the point of tangents! We need to figure them out.

What we know is that the slope of a tangent line is at a is $f'(a)$. Note that $f'(x) = \frac{-1}{(x-1)^2}$ so slope at a is $\frac{-1}{(a-1)^2}$

Tangent line has equation

$$y - f(a) = \frac{-1}{(a-1)^2}(x-a)$$

$$y - \frac{a}{a-1} = \frac{-1}{(a-1)^2}(x-a)$$

Since $(-1, 5)$ is on the line so

$$5 - \frac{a}{a-1} = \frac{-1}{(a-1)^2}(-1-a)$$

$$\frac{5a-5-a}{\cancel{a-1}} = \frac{a+1}{(a-1)^{\cancel{2}}}$$

$$(4a-5)(a-1) = a+1$$

$$4a^2 - 4a - 5a + 5 - a - 1 = 0$$

$$4a^2 - 10a + 4 = 0$$

$$2a^2 - 5a + 2 = 0 \Rightarrow (2a-1)(a-2) = 0$$

$\Rightarrow a = \frac{1}{2}$ and $a = 2$. (the two x-coordinates for the points of tangent)

Then the equations for tangent lines are:

① slope $= f'(\frac{1}{2}) = \frac{-1}{\frac{1}{4}} = -4 \Rightarrow$ $y - 5 = -4(x+1)$ " because $(-1,5)$ is on the line "

② slope $= f'(2) = \frac{-1}{4} \Rightarrow$ $y - 5 = -\frac{1}{4}(x+1)$

MATH 149 Homework 8 Solutions

Section 2.3 #87. $P(t) = 500\left(1 + \frac{4t}{50+t^2}\right)$ 　P: population, t: hours

rate of growth $= P'(t) = 500\left(0 + \frac{4(50+t^2) - 4t \cdot 2t}{(50+t^2)^2}\right)$

$$= 500 \cdot \frac{200-4t^2}{(50+t^2)^2}$$

$P'(2) = 500 \cdot \frac{200 - 4 \cdot 2^2}{(50+2^2)^2} = \frac{500 \cdot 184}{54^2} = 31.55$ bacteria/hour

#96. $f(x) = \frac{x^2+2x-1}{x}$ 　rewrite: $f(x) = x + 2 - x^{-1}$

Then, $f'(x) = 1 + x^{-2}$

$$f''(x) = -2x^{-3} = \frac{-2}{x^3}$$

see the [illegible] file