Section 3.4 # 16. $f(x) = x^3(x-4) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2 \Rightarrow f''(x) = 12x^2 - 24x = 12x(x-2)$

$f'' = 0 \Rightarrow x = 0, x = 2$

		0		2	
f''	+	o	−	o	+
f	concave up		concave down		concave up.

At $x=0$, $x=2$ inflection points

#24. $f(x) = \sin x + \cos x$, $[0, 2\pi]$

$f'(x) = \cos x - \sin x$

$f''(x) = -\sin x - \cos x$

$f''(x) = 0 \Rightarrow \sin x = -\cos x$

$\tan x = -1$

$\Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

	0		$3\pi/4$		$7\pi/4$		2π
$f''(x)$		−	o	+	o	−	
f		concave down		concave up		concave down.	

At $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$, inflection points

34. $g(x) = -\frac{1}{8}(x+2)^2(x-4)^2$

$g'(x) = -\frac{1}{8}\left[2(x+2)(x-4)^2 + (x+2)^2 \cdot 2(x-4)\right]$

$= -\frac{2}{8}(x+2)(x-4)\left(x-4+x+2\right) = -\frac{2}{8}(x^2-2x-8)(2x-2)$

$= -\frac{1}{2}(x^2-2x-8)(x-1)$

$g'(x) = 0 \Rightarrow x = 1, x = 4, x = -2 \rightarrow$ critical numbers.

Second Derivative Test

$g''(x) = -\frac{1}{2}\left[(2x-2)(x-1) + (x^2-2x-8)\cdot 1\right]$

$= -\frac{1}{2}\left[2x^2-3x+2+x^2-2x-8\right] = -\frac{1}{2}\left[3x^2-5x-6\right]$

plug in critical numbers in $g''(x)$ and apply the second derivative test

$g''(1) = -\frac{1}{2}\cdot -8 = 4 > 0 \Rightarrow$ local min at $x=1$

$g''(4) = -\frac{1}{2}(3\cdot 4^2 - 5\cdot 4 - 6) = -11 < 0 \Rightarrow$ local max at $x=4$

$g''(-2) = -\frac{1}{2}\left[3\cdot 4 - 5(-2) - 6\right] = -8 < 0 \Rightarrow$ local max at $x=-2$

Section 3.5 #18. a) highest power x^3

$$\lim_{x\to\infty} \frac{3-2x}{3x^3-1} = \lim_{x\to\infty} \frac{\frac{3}{x^3} - \frac{2}{x^2}}{3 - \frac{1}{x^3}} = \frac{0}{3} = 0$$

(highest power x) b)

$$\lim_{x\to\infty} \frac{3-2x}{3x-1} = \lim_{x\to\infty} \frac{\frac{3}{x} - 2}{3 - \frac{1}{x}} = -\frac{2}{3}$$

highest power x^2 c)

$$\lim_{x\to\infty} \frac{3-2x^2}{3x-1} = \lim_{x\to\infty} \frac{\frac{3}{x^2} - 2}{\frac{3}{x} - \frac{1}{x^2}} = \frac{-2}{0^+} \to -\infty$$

#30. highest power x

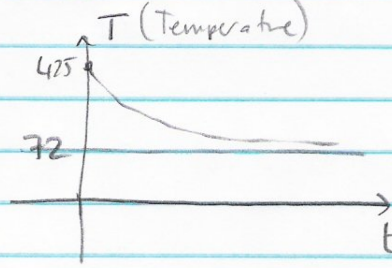
$$\lim_{x\to-\infty} \frac{-3x+1}{\sqrt{x^2+x}} = \lim_{x\to-\infty} \frac{-3 + \frac{1}{x}}{-\sqrt{1+\frac{1}{x}}} = \frac{-3}{-1} = 3$$

Since $x \to -\infty$, $x = -\sqrt{x^2}$

#34.

$$\lim_{x\to\infty} \cos\frac{1}{x} = \cos 0 = 1$$

#88.



$\lim_{t\to 0^+} T = 425°F$ initial temperature of the pie (temperature of the oven)

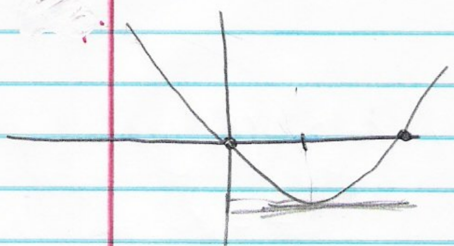
$\lim_{t\to\infty} T = 72°F$ ultimate temperature of the pie (room temperature)

Section 3.4 #56. $f(0) = f(2) = 0$

$f'(x) < 0$ if $x < 1$ → decreasing, $f'(1) = 0 \Rightarrow$ horizontal tangent

$f'(x) > 0$ if $x > 1$ → increasing

$f''(x) > 0 \Rightarrow$ concave up.



#68. Cost $= C = 2x + \frac{300000}{x}$ x: units

To find the minimum, find the critical numbers and make a sign chart for C'.

$$C' = 2 - \frac{300{,}000}{x^2} \Rightarrow C' = 0 \Rightarrow x = \mp\sqrt{150{,}000}$$

C' undefined $\Rightarrow x = 0$

	0		$\sqrt{150{,}000}$		∞
C'	||	+	0	−	
C		increasing		decreasing	

$\Rightarrow$ making a maximum at $x = \sqrt{150{,}000} \cong 387$ units