

Sec 3.7 #8. Find $x, y \in \mathbb{R}^+$ such that $x^2 + y = 27$ and xy maximum

$$x^2 + y = 27 \Rightarrow y = 27 - x^2 \quad f(x) = x \cdot y = x(27 - x^2) =$$

$$f(x) = 27x - x^3$$

$$f'(x) = 27 - 3x^2 = 0 \Rightarrow x = \pm 3 \Rightarrow x = 3 > 0$$

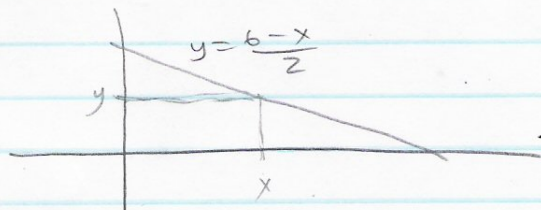
	0	3	
f'		+	-
f		↗	↘

At $x=3$, $f(x)$ is maximum.

$$y = 27 - 3^2 \Rightarrow \boxed{y = 18}$$

Maximum product is $3 \times 18 = 54$

#24.



Area of a rectangle

$$f(x) = x \cdot \frac{6-x}{2} = \frac{6x - x^2}{2}, \quad x \geq 0$$

$$f'(x) = 3 - x$$

$$f'(x) = 0 \Rightarrow x = 3$$

x	0	3	
f'		+	-
f		↗	↘

$$\text{max. at } x=3, y = \frac{6-3}{2} = \frac{3}{2}$$

$$\text{Max Area} = 3 \cdot \frac{3}{2} = \boxed{\frac{9}{2}}$$

#40.



$$V = 3000 \text{ ft}^3 = \frac{4}{3} \pi r^3 + \pi r^2 h$$

$$\text{Surface of cylinder} = 2\pi r h \text{ ft}^2$$

$$\text{Surface of two hemispheres} = 4\pi r^2 \text{ ft}^2$$

cost
 k dollars/ft²
 $2k$ dollars/ft²

Cost: $2k(4\pi r^2) + k \cdot (2\pi r h)$ minimize this function

$$\text{Using } V=3000, \quad h = \frac{3000 - \frac{4}{3}\pi r^3}{\pi r^2} = \frac{3000}{\pi r^2} - \frac{4}{3}r$$

$$\text{The Cost} = f(r) = k \left(8\pi r^2 + 2\pi r \left(\frac{3000}{\pi r^2} - \frac{4}{3}r \right) \right)$$

$$f(r) = k \left[\frac{16}{3}\pi r^2 + \frac{6000}{r} \right]$$

$$f'(r) = k \left[\frac{32}{3}\pi r - \frac{6000}{r^2} \right] = 0 \Rightarrow \boxed{r = \sqrt[3]{\frac{1125}{2\pi}} \approx 5.636 \text{ ft.}}$$

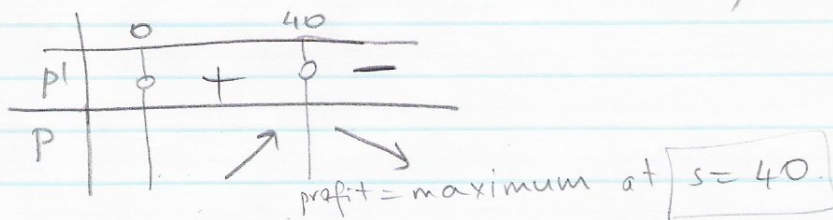
r	0	5.636	∞
f'		-	+
f		↘	↗

min at $r=5.636$, then $h = \boxed{22.545 \text{ ft.}}$

See 3.7 #60a) Profit = $P = -\frac{1}{10}s^3 + 6s^2 + 400$

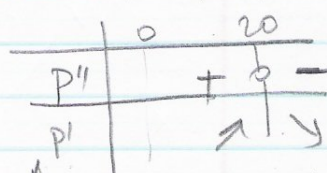
$$P' = -\frac{3}{10}s^2 + 12s = 0 \Rightarrow s(-\frac{3}{10}s + 12) = 0$$

$$s=0, s=40$$



b) To see when the rate of growth of the profit function begins to decline we need to look at P'' .

$$P'' = -\frac{6}{10}s + 12 = 0 \Rightarrow s = 20$$



rate of growth begins to decline when $s=20$.

Sec 3.8 #10

Newton's Method

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$

n	X_n	$f(X_n)$	$f'(X_n)$	$X_n - \frac{f(X_n)}{f'(X_n)}$
1	1	-1	-6	0.8333
2	0.833	-0.1573	-4.1663	0.7955
3	0.7955	-0.0068	-3.7969	0.7937
4	0.7937	0.0000	-3.7718	0.7937

} same