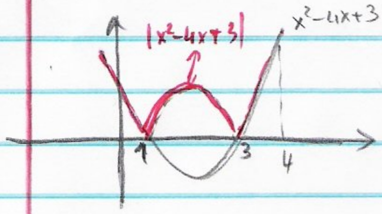
MATH 149 Homework 21 Solutions

Sec 4.4 #26. $\displaystyle\int_0^4 |x^2-4x+3|\,dx = \int_0^1 (x^2-4x+3)\,dx + \int_1^3 -(x^2-4x+3)\,dx + \int_3^4 (x^2-4x+3)\,dx$



$$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_0^1 - \left[\left(\frac{x^3}{3} - 2x^2 + 3x\right)\right]_1^3 + \left[\frac{x^3}{3} - 2x^2 + 3x\right]_3^4$$

$$\boxed{= 4}$$

#52. velocity = (displacement)$'$ $\Rightarrow$ displacement = $\int$ velocity

In particular for this problem distance = $\displaystyle\int_0^5 v(t)\,dt$ is asked
This integral is the area under the curve
of $v(t)$ from $t=0$ to $t=5$.
Counting the number of squares under the curve and noticing that each square has an area of 5, we approximate the distance by $\boxed{29 \text{ squares times } 5 = 145 \text{ ft}}$

#88 $\displaystyle F(x) = \int_{-x}^{x} t^3\,dt = \int_{-x}^{0} t^3\,dt + \int_0^x t^3\,dt = -\int_0^{-x} t^3\,dt + \int_0^x t^3\,dt$

Then

(Second Fundamental Theorem of Calculus) $\Rightarrow F'(x) = -(-x)^3\cdot(-1) + x^3 = -x^3 + x^3 = 0$.

#92. $\displaystyle F(x) = \int_0^{x^2} \sin\theta^2\,d\theta \Rightarrow F'(x) = \sin(x^2)^2 \cdot (x^2)'$

$$= \sin(x^4)\cdot 2x$$

Sec 4.5

#2. $\displaystyle\int x^2\sqrt{x^3+1}\,dx = \int \sqrt{u}\,\frac{du}{3} = \frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$

(substitute $u = x^3+1$
$du = 3x^2\,dx$

$$= \boxed{\frac{2}{9}(x^3+1)^{3/2} + C}$$

#10. $\displaystyle\int \sqrt[3]{(1-2x^2)}\,(-4x)\,dx = \int \sqrt[3]{u}\,du = \frac{3}{4}u^{4/3} + C = \boxed{\frac{3}{4}(1-2x^2)^{4/3} + C}$

(substitute $u = 1-2x^2$
$du = -4x\,dx$

#16. $\displaystyle\int t^3\sqrt{t^4+5}\,dt = \int \sqrt{u}\,\frac{du}{4} = \frac{1}{4}\cdot\frac{2}{3}u^{3/2} + C$

substitute $u = t^4+5$
$du = 4t^3\,dt$

$$= \boxed{\frac{1}{6}(t^4+5)^{3/2} + C}$$