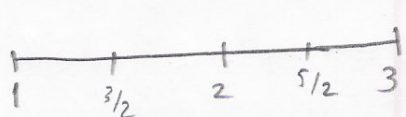


Sec 4.6 #8. $\int_1^3 (4-x^2) dx, n=4$



$f(x) = 4-x^2$

(Trapezoidal Rule) $\int_1^3 (4-x^2) dx \approx \frac{3-1}{2 \cdot 4} (f(1) + 2f(\frac{3}{2}) + 2f(2) + 2f(\frac{5}{2}) + f(3))$
 $\approx \frac{1}{4} ((4-1^2) + 2(4-\frac{9}{4}) + 2(4-2^2) + 2(4-\frac{25}{4}) + (4-3^2)) = -0.7500$

(Simpson's Rule) $\int_1^3 (4-x^2) dx \approx \frac{3-1}{3 \cdot 4} [f(1) + 4f(\frac{3}{2}) + 2f(2) + 4f(\frac{5}{2}) + f(3)] =$
 $= \frac{1}{6} [3 + 4(4-\frac{9}{4}) + 0 + 4(4-\frac{25}{4}) - 5] \approx -0.6667$

Exact $\int_1^3 (4-x^2) dx = [4x - \frac{x^3}{3}]_1^3 = -\frac{2}{3} \approx -0.6667$

#24.

(same interval and same n as in the previous problem)

For the error in Trapezoidal Rule:

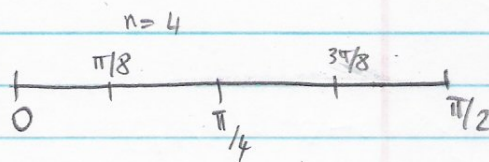
$$|\text{Error}| \leq \frac{(b-a)^3}{12n^2} [\max |f''(x)|]$$

Since $f(x) = 2x+3$, $f'(x) = 2$, $f''(x) = 0$
 So, the $\max |f''(x)| = 0$, hence the error is zero.

Same goes for the Simpson's Rule.

#18.

$\int_0^{\pi/2} \sqrt{1+\cos^2 x} dx$



Trapezoidal Rule: $\approx \frac{\frac{\pi}{2}-0}{2 \cdot 4} (f(0) + 2f(\frac{\pi}{8}) + 2f(\frac{\pi}{4}) + 2f(\frac{3\pi}{8}) + f(\frac{\pi}{2}))$
 $\approx \frac{\pi}{16} (\sqrt{2} + 2\sqrt{1+\cos^2 \frac{\pi}{8}} + 2\sqrt{1+\cos^2 \frac{\pi}{4}} + 2\sqrt{1+\cos^2 \frac{3\pi}{8}} + \sqrt{1+\cos^2 \frac{\pi}{2}})$
 ≈ 1.910

Simpson's Rule $\approx \frac{\frac{\pi}{2}-0}{3 \cdot 4} (f(0) + 4f(\frac{\pi}{8}) + 2f(\frac{\pi}{4}) + 4f(\frac{3\pi}{8}) + f(\frac{\pi}{2}))$
 $\approx \frac{\pi}{24} [\sqrt{2} + 4\sqrt{1+\cos^2 \frac{\pi}{8}} + 2\sqrt{1+\cos^2 \frac{\pi}{4}} + 4\sqrt{1+\cos^2 \frac{3\pi}{8}} + 1]$
 ≈ 1.910

from DERIVE: $\int_0^{\pi/2} \sqrt{1+\cos^2 x} dx \approx 1.910$