MATH 169 Homework 23 Solutions

Sec 4.6 #8. $\int_1^3 (4-x^2)\,dx$, $n=4$ $\qquad f(x)=4-x^2$

$1 \quad 3/2 \quad 2 \quad 5/2 \quad 3$

(Trapezoidal Rule) $\int_1^3 (4-x^2)\,dx \cong \frac{3-1}{2\cdot 4}\left(f(1)+2f\left(\frac{3}{2}\right)+2f(2)+2f(5/2)+f(3)\right)$

$\cong \frac{1}{4}\left((4-1^2)+2\left(4-\left(\frac{3}{2}\right)^2\right)+2\left(4-2^2\right)+2\left(4-\left(\frac{5}{2}\right)^2\right)+\left(4-3^2\right)\right)=\boxed{-0.7500}$

(Simpson's Rule) $\int_1^3 (4-x^2)\,dx \cong \frac{3-1}{3\cdot 4}\left[f(1)+4f(3/2)+2f(2)+4f(5/2)+f(3)\right]=$

$=\frac{1}{6}\left[3+4\left(4-\frac{9}{4}\right)+0+4\left(4-\frac{25}{4}\right)-5\right]\approx\boxed{-0.6667}$

Exact $\int_1^3 (4-x^2)\,dx=\left[4x-\frac{x^3}{3}\right]_1^3=-\frac{2}{3}\approx\boxed{-0.6667}$

#24. (same interval and same n as in the previous problem)

For the error in Trapezoidal Rule:

$$|\text{Error}|\le\frac{(b-a)^3}{12n^2}\left[\max|f''(x)|\right]$$

Since $f(x)=2x+3$, $f'(x)=2$, $f''(x)=0$

So, the $\max|f''(x)|=0$, hence the error is zero.

Same goes for the Simpson's Rule.

#18. $\int_0^{\pi/2}\sqrt{1+\cos^2 x}\,dx$ $\qquad n=4$

$0 \quad \pi/8 \quad \pi/4 \quad 3\pi/8 \quad \pi/2$

Trapezoidal Rule: $\cong \frac{\frac{\pi}{2}-0}{2\cdot 4}\left(f(0)+2f\left(\frac{\pi}{8}\right)+2f\left(\frac{\pi}{4}\right)+2f\left(\frac{3\pi}{8}\right)+f\left(\frac{\pi}{2}\right)\right)$

$\cong\frac{\pi}{16}\left(\underbrace{\sqrt{1+\cos^2 0}}_{\sqrt{2}}+2\sqrt{1+\cos^2\frac{\pi}{8}}+2\sqrt{1+\cos^2\frac{\pi}{4}}+2\sqrt{1+\cos^2\frac{3\pi}{4}}+\underbrace{\sqrt{1+\cos^2\frac{\pi}{2}}}_{1}\right)$

$\cong 1.910$

Simpson's Rule $\cong\frac{\frac{\pi}{2}-0}{3\cdot 4}\left(f(0)+4f\left(\frac{\pi}{8}\right)+2f\left(\frac{\pi}{4}\right)+4f\left(\frac{3\pi}{8}\right)+f\left(\frac{\pi}{2}\right)\right)$

$\cong\frac{\pi}{24}\left[\sqrt{2}+4\sqrt{1+\cos^2\left(\frac{\pi}{8}\right)}+2\sqrt{1+\cos^2\frac{\pi}{4}}+4\sqrt{1+\cos^2\frac{3\pi}{8}}+1\right]$

$\cong 1.910$ $\qquad$ from DERIVE: $\int_0^{\pi/2}\sqrt{1+\cos^2 x}\,dx\cong 1.910$!