MATH 141 Homework 5 Solutions

1.3 #26. $f(x) = 2x^2 - 3x + 1 \quad g(x) = \sqrt[3]{x+6}$

a) $\lim_{x \to 4} f(x) = 2 \cdot 4^2 - 3 \cdot 4 + 1 = 21$

b) $\lim_{x \to 21} g(x) = \sqrt[3]{21+6} = 3$

c) $\lim_{x \to 4} g(f(x)) = g\left(\lim_{x \to 4} f(x)\right) = g(21) = 3.$

#28. $\lim_{x \to \pi} \tan x = \tan \pi = 0.$

#46. $\lim_{x \to -1} \dfrac{2x^2 - x - 3}{x+1} \xrightarrow[x=-1]{\text{plug in}} \dfrac{0}{0}$ needs work!

$$\lim_{x \to -1} \frac{(2x-3)\cancel{(x+1)}}{\cancel{(x+1)}} = \lim_{x \to -1} 2x - 3 = -6$$

#48. $\lim_{x \to -1} \dfrac{x^3+1}{x+1} \to \dfrac{0}{0}$ needs work (Recall $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$)

$$\lim_{x \to -1} \frac{\cancel{(x+1)}(x^2 - x + 1)}{\cancel{(x+1)}} = \lim_{x \to -1} x^2 - x + 1 = 3.$$

#58. $$\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x} = \lim_{x \to 0} \frac{\frac{4-(x+4)}{4(x+4)}}{x} = \lim_{x \to 0} \frac{-\cancel{x}}{4(x+4)} \cdot \frac{1}{\cancel{x}}$$

$$= \lim_{x \to 0} \frac{-1}{4(x+4)} = \frac{-1}{16}.$$

#54. $$\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x} = \lim_{x \to 0} \frac{(\sqrt{2+x} - \sqrt{2})(\sqrt{2+x} + \sqrt{2})}{x(\sqrt{2+x} + \sqrt{2})}$$

$$= \lim_{x \to 0} \frac{\cancel{2} + \cancel{x} - \cancel{2}}{\cancel{x}(\sqrt{2+x} + \sqrt{2})} = \lim_{x \to 0} \frac{1}{\sqrt{2+x} + \sqrt{2}} = \frac{1}{2\sqrt{2}}.$$

#70. $$\lim_{\theta \to 0} \frac{\cos\theta \tan\theta}{\theta} = \lim_{\theta \to 0} \frac{\cancel{\cos\theta} \frac{\sin\theta}{\cancel{\cos\theta}}}{\theta} =$$

$$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1.$$

#78. $$\lim_{x \to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x \to 0} \frac{\sin 2x}{2x} \cdot 2\cancel{x} \cdot \frac{3x}{\sin 3x} \cdot \frac{1}{3\cancel{x}} = \frac{2}{3}$$

1.4 #16. $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \geq 2 \end{cases}$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 - 4x + 6 = 2$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} -x^2 + 4x - 2 = 2$

equal so limit exists.

#28. $f(x) = \begin{cases} x & x < 1 \\ 2 & x = 1 \\ 2x-1 & x > 1 \end{cases}$

The only point that needs to be checked is $x = 1$. At all other points f is continuous.

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} x = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} 2x - 1 = 1$

limit exists: $\lim_{x \to 1} f(x) = 1$

but $f(1) = 2$.

Since $f(1) \neq \lim_{x \to 1} f(x)$, $f(x)$ is not continuous at $x = 1$.

#40. $f(x) = \dfrac{x-3}{x^2-9}$: $f(x)$ is continuous everywhere except $x = 3$ and $x = -3$.

$\lim_{x \to 3} \dfrac{x-3}{x^2-9} = \lim_{x \to 3} \dfrac{x-3}{(x-3)(x+3)} = \dfrac{1}{6}$ but $f(3)$ does not exist.

(removable discontinuity)

$\lim_{x \to -3} \dfrac{x-3}{x^2-9} = \dfrac{-6}{0} \to \infty$ involved

(non removable discontinuity)

#46. $f(x) = \begin{cases} -2x + 3, & x < 1 \\ x^2, & x \geq 1 \end{cases}$

Only point that needs to be checked is $x = 1$.

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} -2x + 3 = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 = 1$

$f(1) = 1^2 = 1$

all equal $\Rightarrow$ $f(x)$ is continuous at $x = 1$

Hence, $f(x)$ is continuous at all real numbers.