Sec-13.2 #22. $\lim\limits_{(x,y)\to(0,0)} \frac{x+y}{x+y^3}$

Look at the limit along the y-axis; that is when $x=0$.

$\lim\limits_{(x,y)\to(0,0)} \frac{0+y}{0+y^3} = \lim\limits_{y\to 0} \frac{1}{y^2} \to \infty$ so limit does not exist

#34. $f(x,y) = \begin{cases} \frac{4x^2y^2}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$ | $g(x,y) = \begin{cases} \frac{4x^2y^2}{x^2+y^2} & (x,y)\neq(0,0) \\ 2 & (x,y)=(0,0) \end{cases}$

Both functions are continuous when $(x,y)\neq 0$

When $(x,y)=(0,0)$ need to find $\lim\limits_{(x,y)\to(0,0)} \frac{4x^2y^2}{x^2+y^2}$

Use polar coordinates: $x=r\cos\theta$, $y=r\sin\theta$ $\Rightarrow = \lim\limits_{r\to 0} \frac{4\cancel{r^2}\cos^2\theta\, r^2\sin^2\theta}{\cancel{r^2}} = \underline{\underline{0}}$

Therefore $f(x,y)$ is continuous at $(0,0)$ whereas $g(x,y)$ is not.

#42. $\lim\limits_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^2} = \lim\limits_{r\to 0} \frac{r\cos\theta\, \cancel{r^2}\sin^2\theta}{\cancel{r^2}} = \underline{\underline{0}}$

#46. $\lim\limits_{(x,y)\to(0,0)} \frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = \lim\limits_{r\to 0} \frac{\sin r}{r} = \underline{\underline{1}}$ (from Calc I or using L'Hospital)

#72 a) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} \underset{\text{along } y=ax}{=} \lim\limits_{x\to 0} \frac{x^2\cdot ax}{x^4+(ax)^2} = \lim\limits_{x\to 0} \frac{ax^3}{x^2(x^2+a^2)}$

$= \lim\limits_{x\to 0} \frac{ax}{x^2+a^2} = 0.$

b) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} \underset{\text{along } y=x^2}{=} \lim\limits_{x\to 0} \frac{x^2\cdot x^2}{x^4+(x^2)^2} = \lim\limits_{x\to 0} \frac{x^4}{2x^4} = \frac{1}{2}$

c) Limit does not exist because the limits along different curves are not equal.

Sec 13.3 #24. $z = \sin 3x \cos 3y \Rightarrow \frac{\partial z}{\partial x} = 3\cos 3x \cos 3y$

$\frac{\partial z}{\partial z} = -3 \sin 3x \sin 3y$

#34. $f(x,y) = \arccos xy$ @ $(1,1)$

$f_x = \frac{-y}{\sqrt{1-(xy)^2}}$ does not exist @ $(1,1)$ b/c denominator is zero.

$f_y = \frac{-x}{\sqrt{1-(xy)^2}}$ does not exist @ $(1,1)$ b/c denominator is zero.

#40. $z = \cos(2x-y)$ @ $x=\frac{\pi}{4}, y=\frac{\pi}{3}$

$\Rightarrow \frac{\partial z}{\partial x} = -2\sin(2x-y)$. @ $(\frac{\pi}{4},\frac{\pi}{3})$, slope $= -2\sin(\frac{2\pi}{4}-\frac{\pi}{3}) = -2\sin(\frac{\pi}{6}) = \boxed{-1}$

$\frac{\partial z}{\partial y} = +\sin(2x-y)$. @ $(\frac{\pi}{4},\frac{\pi}{3})$, slope $= \sin(\frac{\pi}{6}) = \boxed{\frac{1}{2}}$

#46. $f(x,y) = 3x^3 - 12xy + y^3$

$f_x = 9x^2 - 12y = 0 \Rightarrow y = \frac{3}{4}x^2$

$f_y = -12x + 3y^2 = 0 \Rightarrow 4x = y^2$

note that $y \geq 0$, $x \geq 0$

and also $4x = \left(\frac{3}{4}x^2\right)^2 \Rightarrow 64x - 9x^4 = 0$

$x(64 - 9x^3) = 0 \Rightarrow x = 0$ or $x^3 = \frac{64}{9}$, $x = \sqrt[3]{\frac{64}{9}} = \frac{4}{\sqrt[3]{9}}$

$x = 0 \Rightarrow y = 0 \Longrightarrow \boxed{(0,0)}$

$x = \frac{4}{\sqrt[3]{9}} \Rightarrow y = \frac{3}{4}\cdot\left(\frac{4}{\sqrt[3]{9}}\right)^2 = \frac{4}{\sqrt[3]{3}} \Rightarrow \boxed{\left(\frac{4}{\sqrt[3]{9}}, \frac{4}{\sqrt[3]{3}}\right)}$

#76. $f(x,y,z) = \frac{2z}{x+y} \Rightarrow f_x = \frac{-2z}{(x+y)^2}$, $f_{xy} = \frac{4z}{(x+y)^3}$, $f_{xyy} = \frac{-12z}{(x+y)^4}$

$f_y = \frac{-2z}{(x+y)^2} \Rightarrow f_{yx} = \frac{4z}{(x+y)^3} \Rightarrow f_{yxy} = \frac{-12z}{(x+y)^4}$

$f_{yy} = \frac{4z}{(x+y)^3} \Rightarrow f_{yyx} = \frac{-12z}{(x+y)^4}$

all same.

#78. $z = \frac{1}{2}(e^y - e^{-y})\sin x \Rightarrow \frac{\partial z}{\partial x} = \frac{1}{2}(e^y - e^{-y})\cos x$, $\frac{\partial^2 z}{\partial x^2} = \frac{-1}{2}(e^y - e^{-y})\sin x$

$\Rightarrow \frac{\partial z}{\partial y} = \frac{1}{2}(e^y + e^{-y})\sin x$, $+\ \frac{\partial^2 z}{\partial y^2} = \frac{1}{2}(e^y - e^{-y})\sin x$

$\boxed{\text{sum} = 0}$

#100. $U = -5x^2 + xy - 3y^2$

a) $U_x = -10x + y$, $U_y = x - 6y$

b) @ $(2,3)$

$U_x = -20 + 3 = \boxed{-17}$

$U_y = 2 - 6\cdot 3 = \boxed{-16} \Rightarrow$ buy one more of item y.

(because less decrease in satisfaction)