MATH 250 HOMEWORK 12 SOLUTIONS

Sec 13.4 #16 a) $f(x,y) = \frac{x}{y}$ $\quad f(1,2) = \frac{1}{2} = 0.5$, $f(1.05, 2.1) = 0.5$ $\Rightarrow \Delta z = 0$

b) $dz = \frac{1}{y}dx - \frac{x}{y^2}dy$ when $dx = .05$, $dy = .1$ @ $(1,2)$

$dz = \frac{1}{2}(.05) - \frac{1}{2^2}(.1) = 0$

#18. Let $z = f(x,y) = x^2(1+y)^3$

Then $f(2.03, 8.9) - f(2,9) = 2.03^2(1+8.9)^3 - 2^2(1+9)^3 \cong dz$ when $dx = 0.03$, $dy = -0.1$

$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

$dz = 2x(1+y)^3 dx + 3x^2(1+y)^2 dy$

$= 2 \cdot 2(1+9)^3(0.03) + 3 \cdot 2^2(1+9)^2(-0.1) = 0$

$\Rightarrow f(2.03, 8.9) \cong f(2,9) = 2^2(1+9)^3 = 4000$.

#26. $V = \pi r^2 h \Rightarrow dV = 2\pi r h\, dh + \pi r^2 dh$

$\Delta V = \pi(r+dr)^2(h+dh) - \pi r^2 h$ (Note that $dr = \Delta r$, $dh = \Delta h$)

$= \pi r^2 h + \pi h\, dr^2 + 2\pi r h\, dr + \pi r^2 dh + \pi dr^2 dh + 2\pi r\, dr\, dh - \pi r^2 h$

Then $\Delta V - dV = \pi h\, dr^2 + 2\pi r\, dr\, dh$

#46. $f(x,y) = \begin{cases} \frac{5x^2y}{x^3+y^3} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

$f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h} = \frac{0-0}{h} = 0$.

$f_y(0,0) = \lim_{h\to 0} \frac{f(0,0+h) - f(0,0)}{h} = \frac{0-0}{h} = 0$.

So both $f_x(0,0)$ and $f_y(0,0)$ exists.

But $f(x,y)$ is <u>not</u> differentiable at $(0,0)$ b/c

$\lim_{(x,y)\to(0,0)} \frac{5x^2y}{x^3+y^3}$ does not exist. (along $y=x$ limit is $\frac{5}{2}$; along $y=0$ limit is 0) not equal

Section 13.4

#28. $S = \pi r\sqrt{r^2 + h^2}$

$r = 8, h = 20$

$$\frac{dS}{dr} = \pi(r^2 + h^2)^{1/2} + \pi r^2(r^2 + h^2)^{-1/2}$$

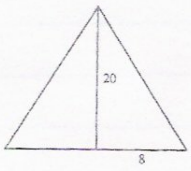
$$= \frac{\pi(r^2 + h^2) + \pi r^2}{(r^2 + h^2)^{1/2}} = \pi\frac{2r^2 + h^2}{\sqrt{r^2 + h^2}}$$

$$\frac{dS}{dh} = \pi r(r^2 + h^2)^{-1/2}h = \pi\frac{rh}{\sqrt{r^2 + h^2}}$$

$$dS = \pi\frac{2r^2 + h^2}{\sqrt{r^2 + h^2}}\,dr + \pi\frac{rh}{\sqrt{r^2 + h^2}}\,dh$$

$$= \frac{\pi}{\sqrt{r^2 + h^2}}[(2r^2 + h^2)\,dr + (rh)\,dh]$$

$S(8, 20) = 541.3758$



Δr	Δh	dS	ΔS	$\Delta S - dS$
0.1	0.1	10.0341	10.0768	0.0427
0.1	−0.1	5.3671	5.3596	−0.0075
0.001	0.002	0.12368	0.12368	0.683×10^{-6}
−0.0001	0.0002	−0.00303	−0.00303	-0.286×10^{-7}