MATH 250 HOMEWORK 14 SOLUTIONS

13.8 #10. $f(x,y) = x^2 + 6xy + 10y^2 - 4y + 4$ relative extrema?

critical points: $f_x = 2x + 6y = 0$, $f_y = 6x + 20y - 4 = 0$ (solve together)

$\Rightarrow x = -6,\ y = 2$

$f_{xx} = 2$, $f_{yy} = 20$, $f_{xy} = 6$. At $(-6,2)$, $f_{xx} > 0$ and $D = f_{xx}f_{yy} - f_{xy}^2 > 0$

So, $(-6,2,0)$ is a relative minimum by Second Partials Test

#26. $f(x,y) = 2xy - \frac{1}{2}(x^4 + y^4) + 1$ relative extrema and saddle?

critical points: $f_x = 2y - 2x^3 = 0$, $f_y = 2x - 2y^3 = 0$

$\Rightarrow y = x^3 \Rightarrow$ $x = y^3 = (x^3)^3 \Rightarrow x = x^9$

$\Downarrow$

$\Rightarrow x - x^9 = 0 \Rightarrow x(x^8 - 1) = 0$

$\Rightarrow x = 0, 1, -1$

So critical points are $(0,0)$, $(1,1)$, $(1,-1)$

$f_{xx} = -6x^2$, $f_{yy} = -6y^2$, $f_{xy} = 2$. $D = f_{xx}f_{yy} - (f_{xy})^2$

At $(0,0)$, $D = -4 < 0 \Rightarrow (0,0,1)$ is a saddle point

At $(1,1)$, $\left.\begin{array}{l} D = -6 \cdot -6 - 2^2 > 0 \\ f_{xx} < 0 \end{array}\right\} \Rightarrow (1,1,2)$ is a relative maximum

At $(-1,-1)$ $\left.\begin{array}{l} D = -6 \cdot (-6) - 2^2 > 0 \\ f_{xx} < 0 \end{array}\right\} \Rightarrow (-1,-1,2)$ is a relative maximum

#44. f has continuous second partial derivatives on an open region containing the critical point (a,b)

If $f_{xx}(a,b)$, $f_{yy}(a,b)$ have opposite signs,

then $D = \underbrace{f_{xx}(a,b)\, f_{yy}(a,b)}_{\text{negative because of opposite sign}} - (f_{xy})^2 < 0$

$\Rightarrow$ f has a saddle point at (a,b).