13.10 #6. Maximize $f(x,y) = x^2 - y^2$ Constraint $g(x,y) = 2y - x^2 = 0$

$\nabla f = \lambda \nabla g \Rightarrow$ $2x = \lambda \cdot -2x \Rightarrow 2x(1+\lambda) = 0 \Rightarrow x = 0$ or $\lambda = -1$

$-2y = \lambda \cdot 2 \Rightarrow y = -\lambda \Rightarrow (0,y)$ and $(x,1)$ are critical.

plug in the critical points in the constraint $(0,y) \Rightarrow 2y - 0^2 = 0 \Rightarrow y = 0 \Rightarrow (0,0)$

$(x,1) \Rightarrow 2 - x^2 = 0 \Rightarrow x = \mp\sqrt{2} \Rightarrow (\sqrt{2}, 1)$, $(-\sqrt{2}, 1)$

$f(0,0) = 0$ minimum, $f(\sqrt{2}, 1) = (\sqrt{2})^2 - 1 = 1$ maximum, $f(-\sqrt{2}, 1) = (-\sqrt{2})^2 - 1 = 1$ maximum

#28. Maximize $f(x,y,z) = z$ subject to $x^2 + y^2 - z^2 = 0$, $x + 2z = 4$ $\Rightarrow \boxed{x = 4 - 2z}$

Intersection of the two constraints: $(4-2z)^2 + y^2 - z^2 = 0$

$16 + 4z^2 - 16z + y^2 - z^2 = 0 = g(y,z)$

$\boxed{3z^2 + 16z + y^2 + 16 = g(y,z) = 0}$

$f(x,y,z) = f(y,z) = z$

$f_y = \lambda g_y \Rightarrow 0 = \lambda \cdot 2y \Rightarrow y = 0$ or $\lambda = 0$

$f_z = \lambda g_z \Rightarrow 1 = \lambda \cdot (6z - 16) \Rightarrow \lambda \neq 0$

Plug in $y = 0$ in the intersection: $3z^2 - 16z + 16 = 0$

$\Rightarrow z = \frac{16 \mp \sqrt{256 - 4 \cdot 3 \cdot 16}}{6} = \frac{16 \mp 8}{6}$ $\Rightarrow z = 4$, $z = \frac{4}{3}$

$z = 4 \Rightarrow x = 4 - 2 \cdot 4 = -4$

$z = \frac{4}{3} \Rightarrow x = 4 - 2 \cdot \frac{4}{3} = 4 - \frac{8}{3} = \frac{4}{3}$

$\Rightarrow (-4, 0, 4)$, $(\frac{4}{3}, 0, \frac{4}{3})$ candidates

$f(-4,0,4) = 4$

$f(\frac{4}{3}, 0, \frac{4}{3}) = \frac{4}{3}$ $\Rightarrow$ maximum is 4 and is attained at $(-4,0,4)$

#42. $P(x,y) = 100 \cdot x^{0.4} y^{0.6}$ maximum P at labor \$48 per unit, capital \$36 per unit is limited to \$100,000

$\boxed{\text{Constraint: } 48x + 36y = \$100{,}000 = g(x,y)}$

$P_x = g_x \Rightarrow 40x^{-0.6}y^{0.6} = 48\lambda \Rightarrow \left(\frac{y}{x}\right)^{0.6} = \frac{48\lambda}{40}$

$P_y = g_y \Rightarrow 60x^{0.4}y^{-0.4} = 36\lambda \Rightarrow \left(\frac{x}{y}\right)^{0.4} = \frac{36\lambda}{60}$

$\Rightarrow \left(\frac{y}{x}\right)^{0.6}\left(\frac{y}{x}\right)^{0.4} = \frac{48\lambda}{40} \cdot \frac{60}{36\lambda}$

$\Rightarrow \frac{y}{x} = 2 \Rightarrow \boxed{y = 2x}$

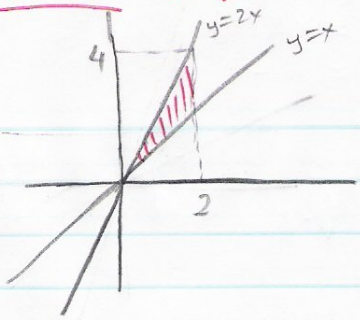
Plug in the constraint: $48x + 36 \cdot 2x = 100{,}000 \Rightarrow 120x = 100{,}000$

$\Rightarrow x = \frac{2500}{3}, y = \frac{5000}{3}$

$P(\frac{2500}{3}, \frac{5000}{3}) \cong \$126.309.71$

#44 similar | $x = 209.65$, $y = 186.35$ $P = 16{,}772$

Sec 14.1 #40. Area between $y=x$, $y=2x$, $x=2$.

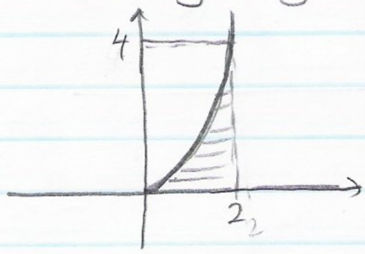


$$\int_0^2\int_x^{2x} 1\,dy\,dx = \int_0^2 y\Big|_x^{2x} dx =$$

$$= \int_0^2 (2x-x)\,dx = \frac{x^2}{2}\Big|_0^2 = 2.$$

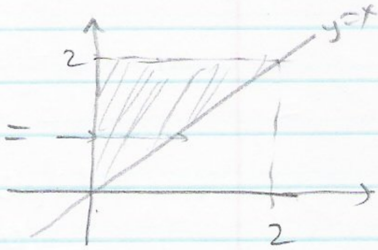
#42. $$\int_0^4 \int_{\sqrt{y}}^2 f(x,y)\,dx\,dy$$

$x=\sqrt{y} \Rightarrow y=x^2$



$$= \int_0^2 \int_0^{x^2} f(x,y)\,dy\,dx$$

#62. $$\int_0^2 \int_x^2 e^{-y^2}\,dy\,dx$$ region =



$$= \int_0^2 \int_0^y e^{-y^2}\,dx\,dy = \int_0^2 \left[e^{-y^2}\,x\right]_0^y dy$$

$$= \int_0^2 e^{-y^2}\cdot y\,dy = \int_{y=0}^{y=2} e^{-u}\cdot -\frac{du}{2}$$

$u=y^2$
$du=2y\,dy$

$$= \left[-\frac{e^{-u}}{-2}\right]_{y=0}^{y=2} = \left[\frac{e^{-y^2}}{2}\right]_0^2 = \frac{e^{-4}}{2} - \frac{e^{-1}}{2} = \frac{e^{-4}-1}{2}$$

$= 0.4908$