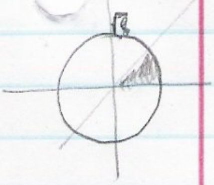
Sec 14.3 #18.

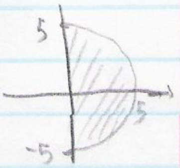
$$\int_0^2 \int_y^{\sqrt{8-y^2}} \sqrt{x^2+y^2}\, dx\, dy = \int_0^{\pi/4} \int_0^{2\sqrt{2}} r^2\, dr\, d\theta$$



$$= \int_0^{\pi/4} d\theta \cdot \int_0^{2\sqrt{2}} r^2\, dr = \theta\Big|_0^{\pi/4} \cdot \frac{r^3}{3}\Big|_0^{2\sqrt{2}} = \boxed{\frac{4\sqrt{2}\pi}{3}}$$

#24.

$$\int_{-\pi/2}^{\pi/2} \int_0^5 e^{-r^2/2}\, r\, dr\, d\theta = \int_{-\pi/2}^{\pi/2} d\theta \cdot \int_0^5 e^{-r^2/2}\, r\, dr\, d\theta$$



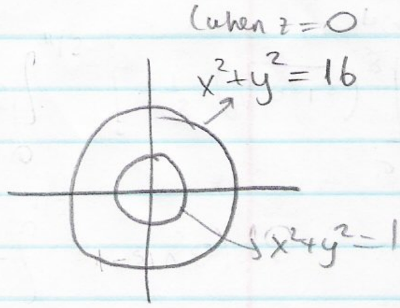

substitute $u = r^2/2$

$du = r\,dr$

$$= \theta\Big|_{-\pi/2}^{\pi/2} \cdot \int_{r=0}^{r=5} e^{-u}\, du = \left(\frac{\pi}{2}+\frac{\pi}{2}\right) \cdot \left[-e^{-u}\right]_{r=0}^{r=5} = \boxed{\pi\left(1-e^{-25/2}\right)}$$

#32.

$\to$ On the xy-plane




So $V = \int_0^{2\pi} \int_1^4 \sqrt{16-r^2}\, r\, dr\, d\theta$ (substitute $u = 16-r^2$)

$$= \int_0^{2\pi} \left[-\frac{1}{3}\left(\sqrt{16-r^2}\right)^3\right]_1^4 d\theta = \int_0^{2\pi} 5\sqrt{15}\, d\theta = \boxed{10\sqrt{15}\pi}$$

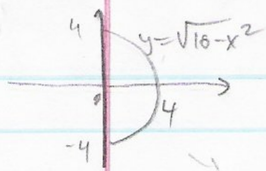
#42.

$$8 \cdot \int_0^{\pi/4} \int_0^{3\cos 2\theta} r\, dr\, d\theta = 4\int_0^{\pi/4} 9\cos^2(2\theta)\, d\theta =$$

$$= 18\int_0^{\pi/4} \left[1+\cos(4\theta)\right] d\theta = 18\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\pi/4} = \boxed{\frac{9\pi}{2}}$$

(half angle formula)

Sec 14.5 #16. $z = 16 - x^2 - y^2 = f(x,y) \Rightarrow f_x = -2x$, $f_y = -2y$




$$dS = \sqrt{1 + f_x^2 + f_y^2}\, dA = \sqrt{1 + 4x^2 + 4y^2}\, dA$$

$$S = \int_0^4 \int_0^{\sqrt{16-x^2}} \sqrt{1 + 4(x^2+y^2)}\, dy\, dx$$

polar coordinates

$$= \int_0^{\pi/2} \int_0^4 \sqrt{1 + 4r^2}\, r\, dr\, d\theta = \frac{\pi}{24}\left(65\sqrt{65} - 1\right)$$

(substitute $u = 1 + 4r^2$)

#38. $f(x,y) = k\sqrt{x^2+y^2} \Rightarrow f_x = \frac{kx}{\sqrt{x^2+y^2}}, \; f_y = \frac{ky}{\sqrt{x^2+y^2}}$

$$dS = \sqrt{1 + f_x^2 + f_y^2}\, dA = \sqrt{1 + \frac{k^2x^2}{x^2+y^2} + \frac{k^2y^2}{x^2+y^2}} = \sqrt{1 + \frac{k^2(x^2+y^2)}{x^2+y^2}} = \sqrt{1+k^2}$$

$$\Rightarrow S = \iint_R \sqrt{1+k^2}\, dA = \sqrt{1+k^2} \iint_R 1\, dA = \sqrt{1+k^2} \cdot \text{Area of } R$$

$$\boxed{= \sqrt{1+k^2} \cdot \pi r^2}$$