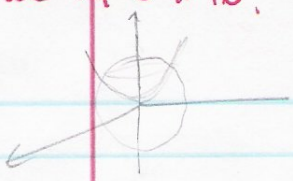
MATH 250 Homework 19 Solutions

Sec 14.6 #16.



$z = \frac{1}{2}(x^2+y^2) \Rightarrow 2z = x^2+y^2 \Rightarrow z \geq 0$

$x^2+y^2+z^2 = 2z+z^2 = 80 \Rightarrow z^2+2z-80=0$

$(z-8)(z+10)$

$\Rightarrow z=8 \Rightarrow x^2+y^2 = 2z = 16$

So, the given integral is

$$\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{\frac{1}{2}(x^2+y^2)}^{\sqrt{80-x^2-y^2}} dz\,dy\,dx$$

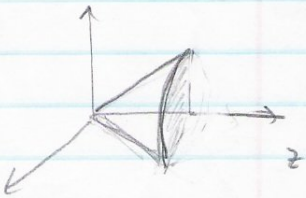
#22.

$$\int_0^2\int_0^{2-x}\int_0^{9-x^2} dz\,dy\,dx = \int_0^2\int_0^{2-x}(9-x^2)\,dy\,dx = \int_0^2 (9-x^2)(2-x)\,dx$$

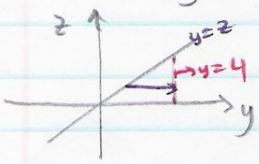
$$= \int_0^2 (18-9x-2x^2+x^3)\,dx = 18x-\frac{9}{2}x^2-\frac{2}{3}x^3+\frac{1}{4}x^4\Big|_0^2 = \boxed{\frac{50}{3}}$$

#26. Elliptic cone: $4x^2+z^2=y^2$

$$\int_0^4\int_z^4\int_0^{\frac{\sqrt{y^2-z^2}}{2}} dx\,dy\,dz$$

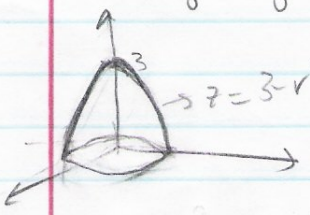


When $x=0$, $z^2=y^2$, $z=\pm y$



14.7 #10

$$\int_0^{2\pi}\int_0^{\sqrt{3}}\int_0^{3-r^2} r\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^{\sqrt{3}} r(3-r^2)\,dr\,d\theta$$



$$= \int_0^{2\pi} d\theta \cdot \int_0^{\sqrt{3}} (3r-r^3)\,dr = \boxed{\frac{9\pi}{2}}$$

#12.

$$\int_0^{2\pi}\int_0^{\pi}\int_2^5 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \int_0^{2\pi} d\theta \cdot \int_0^{\pi}\sin\phi\,d\phi \cdot \int_2^5 \rho^2\,d\rho$$

$$= 2\pi \cdot -\cos\phi\Big|_0^{\pi} \cdot \frac{\rho^3}{3}\Big|_2^5$$

$$= 156\pi$$