Sec 15.2 #58. $\int_C (2x-y)\,dx + (x+3y)\,dy = I$

C: arc on $y = x^{3/2}$ from $(0,0)$ to $(4,8)$

$r(t) = \langle t, t^{3/2} \rangle \quad 0 \le t \le 4 \Rightarrow x = t, \quad y = t^{3/2}$

$dx = dt, \quad dy = \frac{3}{2} t^{1/2} dt$

Then $I = \int_0^4 (2t - t^{3/2})\,dt + (t + 3t^{3/2})\frac{3}{2} t^{1/2}\,dt$

$= \int_0^4 \left(\frac{9}{2}t^2 + \frac{1}{2}t^{3/2} + 2t\right)dt = \left[\frac{3}{2}t^3 + \frac{1}{5}t^{5/2} + t^2\right]_0^4 = \frac{592}{5}$

Sec 15.3 #16. $\int_C \underbrace{(2x-3y+1)}_{M}\,dx \underbrace{-(3x+y-5)}_{N}\,dy$

$\frac{\partial M}{\partial y} = -3 = \frac{\partial N}{\partial x} \Rightarrow \vec{F} = \langle M, N \rangle$ is conservative

and $f(x,y) = x^2 - 3xy - \frac{y^2}{2} + x + 5y + K$ is its potential.

a) $\int_{C_1} M\,dx + N\,dy = \boxed{0}$ b/c C_1 is closed! Same for part (d).

b) $\int_{C_2} M\,dx + N\,dy = \underbrace{f(0,1)}_{\text{terminal}} - \underbrace{f(0,-1)}_{\text{initial}} = \boxed{10}$

c) $\int_{C_3} M\,dx + N\,dy = \underbrace{f(2,e^2)}_{\text{terminal}} - \underbrace{f(0,1)}_{\text{initial}} = \boxed{\frac{1}{2}(3 - 2e^2 - e^4)}$

#20. $\vec{F} = \langle 1, z, y \rangle \Rightarrow \text{curl}\,\vec{F} = 0 \Rightarrow \vec{F}$ is conservative

and its potential is $f = x + yz + K$

a) C_1: initial point: $(\cos 0, \sin 0, 0^2) = (1,0,0)$

terminal point: $(\cos\pi, \sin\pi, \pi^2) = (-1,0,\pi^2)$

$\Rightarrow \int_{C_1} \vec{F}\cdot d\vec{r} = f(-1,0,\pi^2) - f(1,0,0) = \boxed{-2}$

same

b) C_2: initial point: $(1-2\cdot 0, 0, 0) = (1,0,0)$

terminal point $(1-2\cdot 1, 0, \pi^2) = (-1,0,\pi^2)$

$\Rightarrow \boxed{\int_{C_2} F\cdot dr = -2}$

#36. $F = \langle \frac{2x}{y}, \frac{-x^2}{y^2} \rangle$ conservative

similar to above. $\boxed{\text{Work} = \frac{-17}{4}}$