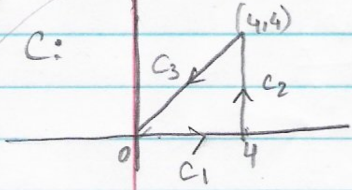
Sec 15.4 #2. $\int_C y^2dx + x^2dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

C:



$C = C_1 + C_2 + C_3$

on C_1: $\langle t, 0\rangle$, $0 \le t \le 4$

on C_2: $\langle 4, t\rangle$, $0 \le t \le 4$

on C_3: $\langle t, t\rangle$, $4 \ge t \ge 0$

$= \int_{C_1} y^2dx + x^2dy + \int_{C_2} y^2dx + x^2dy + \int_{C_3} y^2dx + x^2dy$

$= \int_0^4 0\,dt + t^2 \cdot 0\,dt + \int_0^4 t^2 \cdot 0\,dt + 16\,dt + \int_4^0 t^2dt + t^2dt$

$0 + 16t\Big|_0^4 + \frac{2t^3}{3}\Big|_4^0 = 64 - \frac{128}{3} = \boxed{\frac{64}{3}}$

$M = y^2$ $\quad N = x^2$

$\frac{\partial M}{\partial y} = 2y$ $\quad \frac{\partial N}{\partial x} = 2x$

$\iint_R (2x - 2y)\, dA$

$2\int_0^4 \int_0^x (x-y)\, dy\, dx$

$= 2\int_0^4 \left(xy - \frac{y^2}{2}\right)\Big|_0^x dx$

$= 2\int_0^4 \left(x^2 - \frac{x^2}{2}\right) dx$

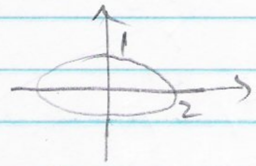
$= \frac{x^3}{3}\Big|_0^4 = \boxed{\frac{64}{3}}$

equal!

#8. $\int_C \underbrace{(y-x)}_{M}dx + \underbrace{(2x-y)}_{N}dy = \iint_R (2-1)\, dA$ = Area of the ellipse

$= \pi \cdot 2 \cdot 1 = \boxed{2\pi}$

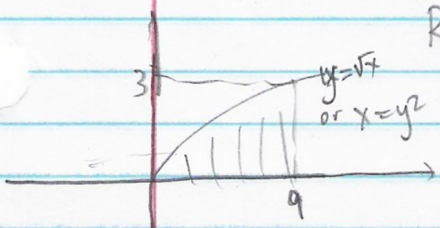
$x = 2\cos\theta$, $y = \sin\theta$ $\Rightarrow$ ellipse



#24. Work Done $= \int_C F \cdot dr = \int_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

$M = 3x^2 + y \Rightarrow \frac{\partial M}{\partial y} = 1$ $\qquad N = 4xy^2 \Rightarrow \frac{\partial N}{\partial x} = 4y^2$

So, Work $= \iint_R (4y^2 - 1)\, dA = \int_0^3 \int_{y^2}^9 (4y^2-1)\, dx\, dy$

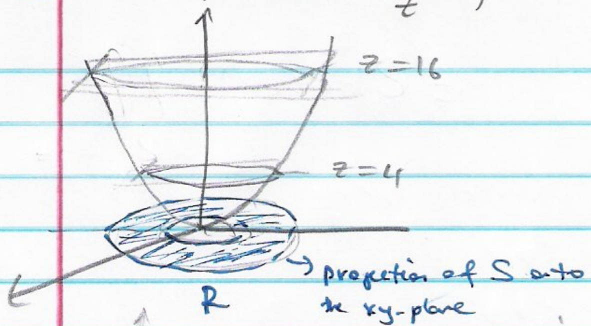


$= \int_0^3 (4y^2-1)\, x\Big|_{y^2}^9 = \int_0^3 (36y^2 - 9 - 4y^4 + y^2)\, dy$

$= \boxed{\frac{558}{5}}$

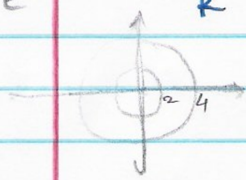
MATH 250 Homework 23 Solutions

Sec 15.6 #18. $f(x,y,z)=\frac{xy}{z}$, S: $z=x^2+y^2$, $4\le x^2+y^2\le 16$



$z=g(x,y)=x^2+y^2 \Rightarrow g_x=2x,\ g_y=2y$

$$\iint_S f\,dS=\iint_R \frac{xy}{z}\sqrt{1+g_y^2+g_x^2}\,dA$$



using polar coordinates (Note that $z=x^2+y^2=r^2$)

$$=\int_0^{2\pi}\int_2^4 \frac{r\cos\theta\, r\sin\theta}{r^2}\sqrt{1+4r^2}\,r\,dr\,d\theta$$

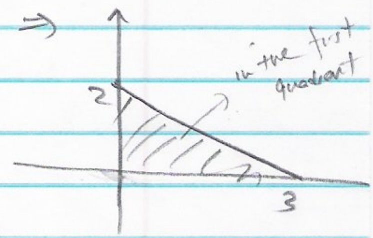
$$\iint_S f\,dS=\left(\int_0^{2\pi}\cos\theta\sin\theta\,d\theta\right)\left(\int_2^4 r\sqrt{1+4r^2}\,dr\right)$$

$\left.\frac{\sin^2\theta}{2}\right|_0^{2\pi}$ $\qquad$ substitute $u=1+4r^2$ $\qquad$ *

$$=0.$$

#24. $F(x,y,z)=x\vec{i}+y\vec{j}=\langle x,y,0\rangle$ $\qquad$ S: $2x+3y+z=6$ first octant ($g(x,y)=z=6-2x-3y$)

projection of S onto the xy plane: (S is a plane) (set $z=0$) $\quad 2x+3y=6 \Rightarrow y=\frac{-2}{3}x+2$



So, $\iint_S F\cdot N\,dS=\iint_R F\cdot\nabla G\,dA$ where

$G(x,y)=z-6+2x+3y$
$(\nabla G=\langle 2,3,1\rangle$

then
$$=\int_0^3\int_0^{-2/3x+2}\langle x,y,0\rangle\cdot\langle 2,3,1\rangle\,dy\,dx$$

$$=\int_0^3\int_0^{-2/3x+2}(2x+3y)\,dy\,dx$$

$$=\int_0^3\left[\frac{-4}{3}x^2+4x+\frac{3}{2}\left(-\frac{2}{3}x+2\right)^2\right]dx=\boxed{12}$$