

MATH 250 Homework 7 Solutions

#17. $x^2 + y^2 = 8x \rightarrow$

in cylindrical coordinates

$$r^2 = 8r \cos \theta \Rightarrow r = 8 \cos \theta$$

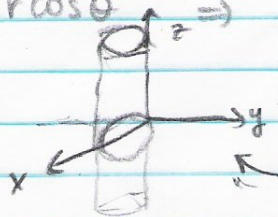
#26. $r = 2 \cos \theta$ in rectangular coordinates:

$$r^2 = 2r \cos \theta \Rightarrow$$

$$x^2 + y^2 = 2x \Rightarrow x^2 - 2x + y^2 = 0$$

$$x^2 - 2x + 1 + y^2 = 1$$

$$(x-1)^2 + y^2 = 1$$



circle in $\mathbb{R}^2$
cylinder in $\mathbb{R}^3$

#44. $x^2 + y^2 - 3z^2 = 0 \Rightarrow$ spherical coordinates

$$(\rho \sin \phi \cos \theta)^2 + (\rho \sin \phi \sin \theta)^2 - 3(\rho \cos \phi)^2 = \rho^2 \sin^2 \phi (\cos^2 \theta + \sin^2 \theta) - 3\rho^2 \cos^2 \phi$$

$$= \rho^2 (\sin^2 \phi - 3\cos^2 \phi) = 0 \Rightarrow \sin^2 \phi = 3\cos^2 \phi \Rightarrow \boxed{\tan^2 \phi = 3}$$

#54. $\rho = 2 \sec \phi \Rightarrow \rho = \frac{2}{\cos \phi} \Rightarrow \rho \cos \phi = 2 \Rightarrow z = 2$

plane parallel to the xy plane

#98. $4(x^2 + y^2) = z^2 \Rightarrow$

cylindrical
 $4r^2 = z^2$

spherical $4(\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta) = \rho^2 \cos^2 \phi$

$$4\rho^2 (\sin^2 \phi (\cos^2 \theta + \sin^2 \theta)) = \rho^2 \cos^2 \phi$$

$$\frac{\sin^2 \phi}{\cos^2 \phi} = \frac{1}{4} \Rightarrow \boxed{\tan \phi = \frac{1}{2}, \tan \phi = -\frac{1}{2}}$$

#100. $x^2 + y^2 = z \Rightarrow \rho^2 \sin^2 \phi = \rho \cos \phi$ (proceed as above)

$\rho = \frac{\cos \phi}{\sin^2 \phi}$ (spherical)

cylindrical

$r^2 = z$

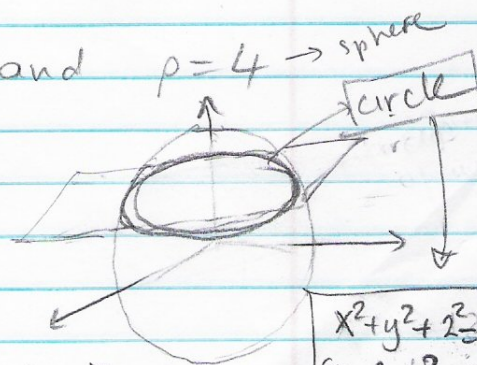
#124. Intersection of $\rho = 2 \sec \phi$ and $\rho = 4 \rightarrow$ sphere

$\rho = \frac{2}{\cos \phi}$

$\Rightarrow \rho \cos \phi = 2$

$\leftarrow z = 2$

plane



$x^2 + y^2 + z^2 = 16$

so $x^2 + y^2 = 12$

OR

$4 = 2 \sec \phi \Rightarrow \cos \phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$