example $\vec{r}(t) = \langle \frac{t^3}{3}, t^2, 2t \rangle$

Find $\vec{T}, \vec{N}, a_T, a_N$ at $t=1$.

$\vec{T} = \frac{r'(t)}{\|r'(t)\|} = \frac{\langle t^2, 2t, 2 \rangle}{|t^2+2|}$

at $t=1$

$\boxed{\vec{T}(1) = \frac{\langle 1, 2, 2 \rangle}{3}}$

Note that it is a unit vector.

$\vec{T}'(t) = \langle \frac{2t(t+2) - t^2 2t}{(t^2+2)^2}, \frac{2(t+2) - 2t \cdot 2t}{(t^2+2)^2}, \frac{-2 \cdot 2t}{(t^2+2)^2} \rangle$

$\vec{T}'(t) = \frac{\langle -2t^3+2t^2+4t, -4t^2+2t+4, -4t \rangle}{(t+2)^2}$

at $t=1$: $T'(1) = \frac{\langle 4, 2, -4 \rangle}{3^2} \rightarrow$ direction $= \langle 2, 1, -2 \rangle$

$N(1) = \frac{\langle 2, 1, -2 \rangle}{\sqrt{9}} = \boxed{\frac{\langle 2, 1, -2 \rangle}{3}}$ → Note that it is a unit vector

$a_T = \vec{a} \cdot \vec{T} = \langle 2, 2, 0 \rangle \cdot \frac{\langle 1, 2, 2 \rangle}{3} = \frac{2}{3} + \frac{4}{3} = \boxed{2}$

$\vec{a} = \langle 2t, 2, 0 \rangle$

at $t=1$ $\vec{a}(1) = \langle 2, 2, 0 \rangle \rightarrow \|\vec{a}\| = \sqrt{4+4} = \sqrt{8}$

$a_N = \sqrt{\|\vec{a}\|^2 - a_T^2} = \sqrt{8 - 2^2} = \boxed{2}$.

Check $\vec{a} = 2 \cdot \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle + 2 \langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \rangle$

$= \langle \frac{6}{3}, \frac{6}{3}, \frac{0}{3} \rangle = \langle 2, 2, 0 \rangle$ ✓