

Sec 13.1 #18. $f(x,y) = \sqrt{4-x^2-4y^2}$

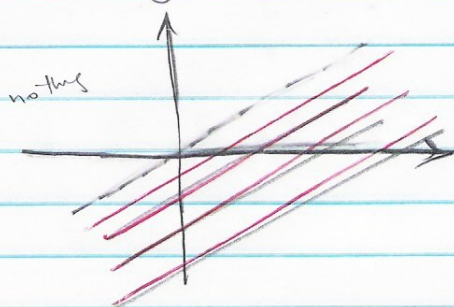
Domain: $4-x^2-4y^2 \geq 0 \Rightarrow x^2+4y^2 \leq 4 \Rightarrow \frac{x^2}{4} + \frac{y^2}{1} \leq 1$ (inside an ellipse)

Range: $0 \leq z \leq 2$ (max value for $4-x^2-4y^2$ is $4-0=4$ and the square root is non-negative)

#20. $f(x,y) = \arccos \frac{y}{x}$

Domain = $\{(x,y) \mid -1 \leq \frac{y}{x} \leq 1\}$, Range: $0 \leq z \leq \pi$

#56. $f(x,y) = \ln(x-y)$ level curves: $k = \ln(x-y)$



$e^k = x-y \Rightarrow y = x - e^k$
 parallel lines
 of slope 1
 & y-intercept negative

#72. $f(x,y,z) = x^2 + \frac{1}{4}y^2 - z$
 level surface at $c=1$ is

$1 = x^2 + \frac{y^2}{4} - z$

elliptic paraboloid

#78. $V(x,y) = \frac{5}{\sqrt{25+x^2+y^2}}$

level curves: $k = \frac{5}{\sqrt{25+x^2+y^2}}$

$25+x^2+y^2 = \frac{25}{k^2}$

$x^2+y^2 = \frac{25}{k^2} - 25$

circle when $|k| < 1$
 point when $|k| = 1$
 empty when $|k| > 1$

#80. $z = C x^a y^{1-a}$

$\ln z = \ln C + \ln x^a + \ln y^{1-a}$

$= \ln C + a \ln x + (1-a) \ln y$

$\ln z - \ln y = \ln C + a (\ln x - \ln y)$

$\ln \frac{z}{y} = \ln C + a \ln \frac{x}{y}$