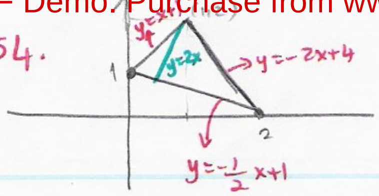
MATH 250 HOMEWORK 15 SOLUTIONS

13.8 #54.



$f(x,y) = (2x-y)^2$

$f_x = 4(2x-y) = 0 \Rightarrow y = 2x$

$f_y = -2(2x-y) = 0 \Rightarrow y = 2x$

On $y = x+1$, $0 \le x \le 1$, $f(x,y) = f(x) = (2x-(x+1))^2 = (x-1)^2$

maximum 1 at $x=0$, $\boxed{\text{minimum } 0 \text{ at } x=1}$

On $y = -\frac{1}{2}x+1$, $0 \le x \le 2$, $f(x,y) = f(x) = \left(2x-\left(-\frac{1}{2}x+1\right)\right)^2 = \left(\frac{5}{2}x-1\right)^2$

maximum is $\boxed{16 \text{ at } x=2}$, minimum is 0 at $x = \frac{2}{5}$

On $y = -2x+1$, $1 \le x \le 2$, $f(x,y) = f(x) = (2x-(-2x+4))^2 = (4x-4)^2$

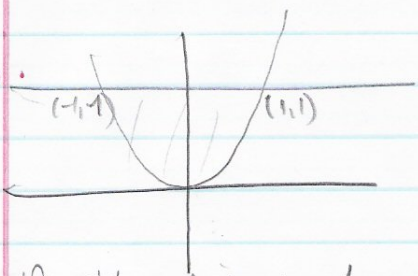
$\boxed{\text{maximum is } 16 \text{ at}}$ $x=2$, $\boxed{\text{minimum is } 0 \text{ at } x=1}$.

At the critical points when $y=2x$, $f(x,y) = (2x-2x)^2 = 0$

So, $f(x,y)$ has absolute max of 16 at $(2,0)$

and absolute min of 0 at $(1,2)$ and along $y=2x$.

#56.



$f(x,y) = 2x - 2xy + y^2$

$f_x = 2 - 2y = 0 \Rightarrow y = 1$

$f_y = 2y - 2x = 0 \Rightarrow y = x$

$(1,1)$ critical. $\boxed{f(1,1) = 1}$

On the line $y=1$, $-1 \le x \le 1$, $f(x,y) = f(x) = 2x - 2x + 1 = 1$

On the curve $y = x^2$, $-1 \le x \le 1$,

$f(x,y) = f(x) = 2x - 2x(x^2) + (x^2)^2 = x^4 - 2x^3 + 2x$

$f'(x) = 4x^3 - 6x^2 + 2 = 2(2x^3 - 3x^2 + 1) = 0 \Rightarrow 2(x-1)^2(2x+1) = 0$

$\Rightarrow x = -\frac{1}{2}, x = 1$

$f(-\frac{1}{2}) = -\frac{11}{16}$, $f(-1) = 1$, $f(1) = 1$

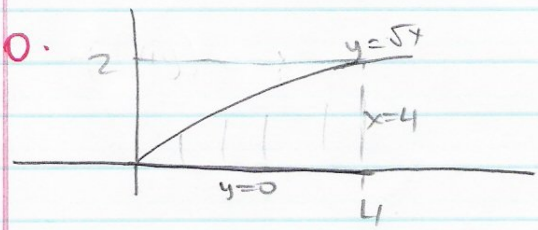
end points

(can use DERIVE or Mathematica to factor)

So, absolute max is 1 at $(1,1)$ and on $y=1$.

absolute min is $-\frac{11}{16}$ at $\left(-\frac{1}{2}, \frac{1}{4}\right)$

#60.



$f(x,y) = x^2 - 4xy + 5$

$f_x = 2x - 4y = 0$

$f_y = -4x = 0$

$\rightarrow x = y = 0$.

$f(0,0) = 5$

Along $y=0$, $0 \le x \le 4$, $f(x) = x^2+5$ max: 21 at $x=4$, min: 4 at $x=0$

Along $x=4$, $0 \le y \le 2$, $f(y) = 16 - 16y + 5$

$f(0) = 5$, $f(2) = -11$ $\rightarrow$ max 5 at $y=0$, min -11 at $y=2$.

Along $y = \sqrt{x}$, $0 \le x \le 4$, $f = x^2 - 4x^{3/2} + 5 \Rightarrow f'(x) = 2x - 6x^{1/2} \neq 0$ on $[0,4]$

So, the maximum is $f(4,0) = 21$ and the minimum is $f(4,2) = -11$