MATH 250 Homework 21 Solutions

Sec 15.1 #38. $\vec{F}(x,y) = \underbrace{3x^2y^2}_{M} i + \underbrace{2x^3y}_{N} \vec{J}$ $\quad M_y = 6x^2y$, $N_x = 6x^2y$.

equal so F is conservative

Need to find f such that $f_x = 3x^2y^2$ and $f_y = 2x^3y$.

$f_x = 3x^2y^2 \Rightarrow f = \int 3x^2y^2\,dx = x^3y^2 + g(y)$

Then $f_y = 2x^3y + g'(y) = 2x^3y \Rightarrow g'(y) = 0 \Rightarrow g(y) = K$ (constant)

$\Rightarrow f(x,y) = x^3y^2 + K$ and $\nabla f = \vec{F}$

#52. $\vec{F}(x,y,z) = e^z(y\vec{i} + x\vec{J} + \vec{k})$

$$\text{curl}\,\vec{F} = \begin{vmatrix} i & J & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^z y & e^z x & e^z \end{vmatrix} = \left(\frac{\partial}{\partial y}e^z - \frac{\partial}{\partial z}e^z x\right)\vec{i} - \left(\frac{\partial e^z}{\partial x} - \frac{\partial e^z y}{\partial z}\right)\vec{J} + \left(\frac{\partial e^z x}{\partial x} - \frac{\partial e^z y}{\partial y}\right)\vec{k}$$

$$= e^z x\vec{i} + e^z y\vec{J} + (e^z - e^z)\vec{k} \neq \vec{0}$$

So $\vec{F}$ is not conservative.

#54. $\vec{F}(x,y,z) = y^2z^3\vec{i} + 2xyz^3\vec{J} + 3xy^2z^2\vec{k}$

Verify that $\text{curl}\,\vec{F} = \vec{0}$ so F is conservative.

Need to find f such that $f_x = y^2z^3$, $f_y = 2xyz^3$, $f_z = 3xy^2z^2$

$f = \int y^2z^3\,dx = xy^2z^3 + g(y,z)$

$f_y = 2xyz^3 + g_y(y,z) = 2xyz^3 \Rightarrow g_y(y,z) = 0$

$g(y,z) = h(z)$

So, $f = xy^2z^3 + h(z)$

$f_z = 3xy^2z^2 + h'(z) = 3xy^2z^2$

$\Rightarrow h'(z) = 0 \Rightarrow h(z) = K$ constant

$\Rightarrow f = xy^2z^3 + K$ and $\nabla f = \vec{F}$.

Sec 15.2 #10. $\int_C 8xyz\,ds = \int_0^2 8\cdot 12t\cdot 5t\cdot 3\,(13\,dt) = \boxed{49920}$

$C: r(t) = \langle 12t, 5t, 3\rangle \Rightarrow r'(t) = \langle 12, 5, 0\rangle$

$0 \le t \le 2 \Rightarrow |r'(t)| = 13$

#14. $\int_C x^2+y^2\,ds = \int_0^{\pi/2} (4\cos^2 t + 4\sin^2 t)\cdot 2\,dt = \int_0^{\pi/2} 8\,dt = \boxed{4\pi}$

$r(t) = \langle 2\cos t, 2\sin t\rangle \Rightarrow r'(t) = \langle -2\sin t, 2\cos t\rangle$

$0 \le t \le \frac{\pi}{2}$ $\quad |r'(t)| = 2$

#24. $C: r(t) = t^2\vec{i} + 2t\vec{j}$ $\quad \rho(x,y) = \frac{3}{4}y$

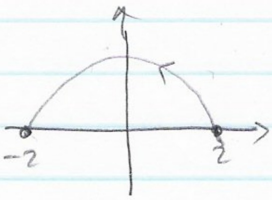
$0 \le t \le 1$

$r'(t) = \langle 2t, 2\rangle$ $\quad$ mass $= \int_C \rho(x,y)\,ds = \int_0^1 \frac{3}{4}\cdot 2t\cdot\sqrt{4t^2+4}\,dt$

$|r'(t)| = \sqrt{4t^2+4}$

$= 3\int_0^1 t\sqrt{t^2+1}\,dt = 2\sqrt{2}-1$ $\quad$ (substitute $u = t^2+1$)

#38. $\vec{F}(x,y) = \langle -y, -x\rangle$



$0 \le t \le \pi$

$r(t) = \langle 2\cos t, 2\sin t\rangle$

$r'(t) = \langle -2\sin t, 2\cos t\rangle$

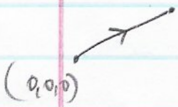
$\vec{F} = \langle -2\sin t, -2\cos t\rangle$

Work $= \int F\cdot r'(t)\,dt = \int_0^\pi \langle -2\sin t, -2\cos t\rangle\cdot\langle -2\sin t, 2\cos t\rangle\,dt$

$= \int_0^\pi (4\sin^2 t - 4\cos^2 t)\,dt = -4\int_0^\pi \cos^2 t - \sin^2 t\,dt = -4\int_0^\pi \cos 2t\,dt$

$= -2\sin 2t\Big|_0^\pi = \boxed{0}$

#40. $\vec{F}(x,y,z) = \langle yz, xz, xy\rangle = \langle 6t^2, 10t^2, 15t^2\rangle$



$r(t) = \langle 5t, 3t, 2t\rangle,\ 0 \le t \le 1$ $\quad$ Work $= \int_0^1 F\cdot r'(t)\,dt$

$r'(t) = \langle 5, 3, 2\rangle$

$= \int_0^1 (30t^2 + 30t^2 + 30t^2)\,dt = \boxed{30}$