MATH 250 Homework 4 Solutions.

11.3 #58. If $\left\|\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v}\right\| = \left\|\frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\cdot\vec{u}\right\|$ then

$$\frac{\|\vec{u}\cdot\vec{v}\|}{\|\vec{v}\|^2}\cdot\|\vec{v}\| = \frac{\|\vec{u}\cdot\vec{v}\|}{\|\vec{u}\|^2}\cdot\|\vec{u}\| \Rightarrow \frac{1}{\|\vec{v}\|} = \frac{1}{\|\vec{u}\|} \Rightarrow \|\vec{v}\| = \|\vec{u}\|$$

So yes, unless $\vec{u}$ or $\vec{v}$ is the zero vector.

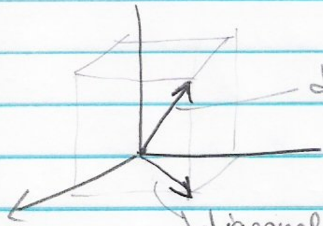
#76. $\vec{u},\vec{v} \perp \vec{w} \Rightarrow \vec{u}\cdot\vec{w} = 0$ and $\vec{v}\cdot\vec{w} = 0$

Now, $(\vec{u}+\vec{v})\cdot\vec{w} = \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w}$ (distributive property of dot product over +)

$= 0 + 0 = 0$

So, yes $\vec{u}+\vec{v} \perp \vec{w}$.

#78.



diagonal of the cube $\langle 1,1,1\rangle$

diagonal of the side $\langle 1,1,0\rangle$

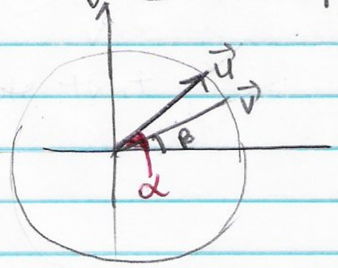
$$\cos\theta = \frac{\langle 1,1,1\rangle\cdot\langle 1,1,0\rangle}{\sqrt{3}\cdot\sqrt{2}}$$

$$\cos\theta = \frac{2}{\sqrt{6}}$$

$$\Rightarrow \theta = 35.26^\circ.$$

#86. $\vec{u} = \langle\cos\alpha, \sin\alpha, 0\rangle$ these two vectors can

$\vec{v} = \langle\cos\beta, \sin\beta, 0\rangle$ be considered on the xy-plane only b/c $z=0$.



Note that $\|\vec{u}\|=1$, $\|\vec{v}\|=1$ and angle between $\vec{u}$ and $\vec{v}$ is $\alpha-\beta$.

$$\cos(\alpha-\beta) = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

#88. Cauchy-Schwarz Inequality: $|u\cdot v| \le \|u\|\cdot\|v\|$

Recall $|u\cdot v| = \big|\|u\|\cdot\|v\|\cdot\cos\theta\big| = \|u\|\,\|v\|\cdot|\cos\theta| \le \|u\|\cdot\|v\|\cdot 1$

because $|\cos\theta| \le 1$

So, $|u\cdot v| \le \|u\|\cdot\|v\|$.