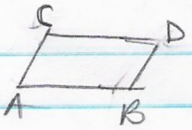
11.4 #32 $A(2,-3,1)$, $B(6,5,-1)$, $C(3,-6,4)$, $D(7,2,2)$

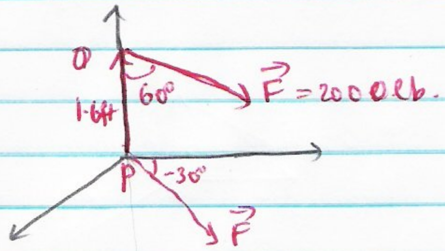
$\vec{AB} = \langle 4,8,-2\rangle$, $\vec{CD} = \langle 4,8,-2\rangle \Rightarrow \vec{AB} \parallel \vec{CD}$

$\vec{AC} = \langle 1,-3,3\rangle$, $\vec{BD} = \langle 1,-3,3\rangle \Rightarrow \vec{AC} \parallel \vec{BD}$



Area $= \|\vec{AB}\times\vec{AC}\| = 2\sqrt{230}$

#38.



$\vec{PQ} = 0.16\ \langle 0,0,1\rangle$

$\vec{F} = 2000\ \langle 0, \cos(-30), \sin(-30)\rangle$

$\vec{F} = 2000\ \langle 0, \frac{\sqrt{3}}{2}, -\frac{1}{2}\rangle$

$$\text{Torque} = M = \vec{PQ}\times\vec{F} = \begin{vmatrix} i & j & k \\ 0 & 0 & 0.16 \\ 0 & 1000\sqrt{3} & -1000 \end{vmatrix} = -160\sqrt{3}\, i$$

$$\|\vec{PQ}\times\vec{F}\| = 160\sqrt{3}\ \text{ft-lb}$$

#46. Volume of parallelepiped $|u\cdot(\vec{v}\times\vec{w})| = \left|\begin{vmatrix} 1 & 3 & 1 \\ 0 & 6 & 6 \\ -4 & 0 & -4 \end{vmatrix}\right| = |-72| = 72.$

#56. Prove $u\cdot(v\times w) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

LHS $= u\cdot\langle v_2w_3 - v_3w_2, -(v_1w_3 - v_3w_1), v_1w_2 - v_2w_1\rangle$

$u_1\cdot(v_2w_3 - v_3w_2) - u_2(v_1w_3 - v_3w_1) + u_3(v_1w_2 - v_2w_1)$

RHS $= u_1(v_2w_3 - v_3w_2) - u_2(v_1w_3 - v_3w_1) + u_3(v_1w_2 - v_2w_1)$) equal

(det expanded with respect to 1st row)

#60. Prove $u\cdot(v\times w) = (u\times v)\cdot w$

From #56, LHS $= u_1v_2w_3 - u_1v_3w_3 - u_2v_1w_3 + u_2v_3w_1 + u_3v_1w_2 - u_3v_2w_1$

RHS $(u\times v)\cdot w = \langle u_2v_3 - u_3v_2, -(u_1v_3 - u_3v_1), u_1v_2 - u_2v_1\rangle\cdot\langle w_1, w_2, w_3\rangle$

$= u_2v_3w_1 - u_3v_2w_1 - u_1v_3w_2 + u_3v_1w_2 + u_1v_2w_3 - u_2v_1w_3$

Notice that LHS = RHS.

11.5 #18. Point $(2,1,2)$

direction $\langle -1, 1, 1\rangle$ (b/c parallel to the given line)

$$\Rightarrow \begin{cases} x = -t+2 \\ y = t+1 \\ z = t+2 \end{cases}$$

#28. $x = -3t+1,\ y = 4t+1,\ z = 2t+4$

$x = 3s+1,\ y = 2s+4,\ z = -s+1$

Intersect? $\begin{cases} -3t+1 = 3s+1 \\ 4t+1 = 2s+4 \\ 2t+4 = -s+1 \end{cases}$ $\Rightarrow t = -s$

$\Rightarrow -4s+1 = 2s+4 \Rightarrow -6s = 3 \Rightarrow s = -\frac{1}{2}$

then $t = \frac{1}{2}$

plug in $t = \frac{1}{2}$, $s = -\frac{1}{2}$: $2\cdot\left(\frac{1}{2}\right)+4 = -\frac{1}{2}+1$ Not true!

So the two lines do <u>NOT</u> intersect.

#42. Plane containing $P(2,3,-2)$, $Q(3,4,2)$, $R(1,-1,0)$

normal $= \vec{PQ} \times \vec{QR} = \langle 1,1,4\rangle \times \langle -1,-4,2\rangle$

$= \langle 18,-6,-3\rangle = -3\underbrace{\boxed{\langle -6,2,1\rangle}}_{\text{normal}}$

Equation of the plane is (using the point P)

$$-6(x-2) + 2(y-3) + (z+2) = 0$$

$$-6x + 2y + z + 8 = 0.$$

#50. Plane containing $P(3,2,1)$ and $Q(3,1,-5)$ and perpendicular to the plane $6x+7y+2z=10$.

normal $= \vec{n} \perp \langle 6,7,2\rangle$ and $\vec{n} \perp \vec{PQ} = \langle 0,-1,-6\rangle$ $\Rightarrow \vec{n} = \langle 6,7,2\rangle \times \langle 0,-1,-6\rangle$

$\Rightarrow \vec{n} = \langle -40, 36, -6\rangle$

$= -2\underbrace{\langle 20,-18,3\rangle}_{\text{normal}}$

Equation of the plane: (using the point P)

$$20(x-3) - 18(y-2) + 3(z-1) = 0.$$

$$20x - 18y + 3z - 27 = 0.$$