Math 250 Homework 6 Solutions

11.5 #60. Determine $\parallel$, $\perp$ or neither

plane 1: $3x+2y-z=7$ normal $=\vec{n}_1=\langle 3,2,-1\rangle$

plane 2: $x-4y+2z=0$ normal $=\vec{n}_2=\langle 1,-4,2\rangle$

n_1 & n_2 are not parallel.

$\vec{n}_1\cdot\vec{n}_2 = 3-8-2=-7 \Rightarrow$ not orthogonal

$\cos\theta = \frac{\vec{n}_1\cdot\vec{n}_2}{\|n_1\|\cdot\|n_2\|} = \frac{-7}{\sqrt{14}\cdot\sqrt{21}} = \frac{-1}{\sqrt{6}}$

$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{6}}\right) \approx 114^\circ$

#82. Intersection of $6x-3y+z=5$ and $-x+y+5z=5$

Direction of the line of intersection: cross product of normals $=\langle 6,-3,1\rangle\times\langle -1,1,5\rangle$
$=\langle -16,-31,3\rangle$

To find a point in the intersection set $z=0$ in both equations. (or any other point)

So $\left.\begin{aligned}6x-3y&=5\\-x+y&=5\end{aligned}\right\} \Rightarrow y=\frac{35}{3},\ x=\frac{20}{3} \Rightarrow \left(\frac{20}{3},\frac{35}{3},0\right)$ on the line

$$\Rightarrow \boxed{x=\frac{20}{3}-16t \qquad y=\frac{35}{3}-31t \qquad z=3t}$$

#84. $2x+3y=-5$, $\frac{x-1}{4}=\frac{y}{2}=\frac{z-3}{6} \Rightarrow \left.\begin{aligned}x&=4t+1\\y&=2t\\z&=6t+3\end{aligned}\right\}$ plug in the equation of the plane

$2(4t+1)+3\cdot 2t=-5 \Rightarrow 8t+2+6t=-5 \Rightarrow t=-\frac{1}{2}$

So the intersection point is $x=-1$, $y=-1$, $z=0$.
(plug in $t=-\frac{1}{2}$ in the parametric line equation)

The line is not on the plane b/c if it were the solution would be all points on the line.

#90 Distance from $(3,2,1)$ to $x-y+2z=4$ using the formula we derived in class is $\frac{|3-2+2\cdot1-4|}{\sqrt{1^2+(-1)^2+2^2}} = \boxed{\frac{1}{\sqrt{6}}}$

#92. $\left.\begin{aligned}4x-4y+9z&=7\\4x-4y+9z&=18\end{aligned}\right\}$ normals are the same so parallel

Distance: pick a point on one of the planes and find its distance to the other one.

Let $x=0$, $y=0$ in the second plane equation. Then $z=2$.

Distance from $(0,0,2)$ to the first plane is: $\frac{|0-0+18-7|}{\sqrt{16+16+81}} = \frac{11}{\sqrt{113}}$